A GAME-THEORETIC MODEL FOR REASONABLE ROYALTY CALCULATION‡

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ABSTRACT

This article describes a model for a hypothetical licensing negotiation for determining a reasonable royalty according to 35 U.S.C. § 284 as interpreted by Georgia-Pacific Corp. v. United States Plywood Corp. It connects the fifteen Georgia-Pacific factors to game-theoretic bargaining solutions and microeconomic market models. The result is a comprehensive framework for a systematic reasonable royalty analysis.

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INTRODUCTION

The first part (I.) of this article briefly describes the hypothetical approach for determining a reasonable royalty according to 35 U.S.C. § 284, as interpreted by *Georgia-Pacific Corp. v. United States Plywood Corp.*[1] The second part (II.) introduces game-theoretic bargaining solutions, such as the Nash and Rubinstein bargaining solution, for modeling the framework of the hypothetical negotiation. In the third part (III.), the details of the hypothetical negotiation and the individual elements of the Nash bargaining solution are addressed showing their correspondence to the *Georgia-Pacific* factors. The outcome of the hypothetical negotiation is then discussed in the fourth part (IV.). Finally, in the fifth part (V.), the article concludes with a

[1] 318 F.Supp. 1116 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

summary of the established model.

I. HYPOTHETICAL APPROACH

Title 35 U.S.C. § 284 determines the amount of damages an infringer of a patent has to pay to the claimant.[2] The statute states that "[u]pon finding for the claimant the court shall award the claimant damages adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer . . . ."[3] Accordingly, in the first place, the claimant is entitled to *lost profits*, that is, money to compensate for the profit the claimant lost as a result of the infringement.[4] However, if the claimant cannot prove lost profits, he can demand a *reasonable royalty* for the use made of the invention by the infringer.[5]

As opposed to lost profits, which refer to the claimant's profit, a reasonable royalty is based on the infringer's profit.[6] But the reasonable royalty determination is not identical to simply calculating the infringer's profit.[7] While it is true that the infringer's profit is the basis for the reasonable royalty analysis under the hypothetical approach, the royalty ultimately depends on how a willing licensor and a willing licensee in a hypothetical negotiation would have agreed to distribute the profit.[8] Thus, for

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[2] 35 U.S.C.A. § 284 (West, Westlaw through P.L. 112-89 (excluding P.L. 112-55, 112-74, 112-78, and 112-81) approved 1/3/12). Similar provisions can be found in intellectual property laws of various other countries. *See, e.g.*, Gesetz über Urheberrecht und verwandte Schutzrechte [Urheberrechtsgesetz] [UrhG] [Copyright Act], Sept. 9, 1965, BGBL. I at 1273, art. 97(2), as amended (Ger.), *available at* http://www.gesetze-im-internet.de/urhg/BJNR012730965.html#BJNR012730965BJNG000101377; [Patentgesetz] [PatG] [Patent Act], Dec. 16, 1980, BGBL. I at 1, § 139(2), as amended (Ger.), *available at* http://www.gesetze-im-internet.de/patg/BJNR201170936.html#BJNR201170936BJNG000100311; Gesetz über den Schutz von Marken und sonstigen Kennzeichen [Markengesetz] [MarkenG] [Trademark Act], Oct. 25, 1994, BGBL. I at 3082, § 14(6), as amended (Ger.), *available at* http://www.gesetze-im-internet.de/markeng/BJNR308210994.html; Tokkyoh [Patent Law], Law No. 121 of 1959 art. 102(2), as amended (Japan). Thus, the general ideas of this article are applicable beyond 35 U.S.C. 284.

[3] 35 U.S.C.A. § 284 (West, Westlaw through P.L. 112-89 (excluding P.L. 112-55, 112-74, 112-78, and 112-81) approved 1/3/12).

[4] *Georgia-Pacific*, 318 F. Supp. at 1127.

[5] *Id.*

[6] Barry L. Grossman, *Patent Infringement Damages*, *in* PATENT LITIGATION STRATEGIES HANDBOOK 1403, 1405–06, 1418 (Barry L. Grossman & Gary M. Hoffman eds., 3d ed. 2010).

[7] *See Georgia-Pacific*, 318 F. Supp. 1121 (discussing the elements used in determining the reasonable royalty).

[8] Radio Steel & Mfg. Co. v. MTD Prods, Inc., 788 F.2d 1554, 1557 (Fed. Cir.

determining a reasonable royalty by means of the hypothetical approach, the calculation of the infringer's profit must be embedded into a hypothetical negotiation.

In *Georgia-Pacific*, the court enumerated fifteen factors for evaluating a hypothetical negotiation.[9] In factor 15 the court defined a reasonable royalty as

> [t]he amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time the infringement began) if both had been reasonably and voluntarily trying to reach an agreement; that is, the amount which a prudent licensee—who desired, as a business proposition, to obtain a license to manufacture and sell a particular article embodying the patented invention—would have been willing to pay as a royalty and yet be able to make a reasonable profit and which amount would have been acceptable by a prudent patentee who was willing to grant a license.[10]

The hypothetical reasonable royalty negotiation described by *Georgia-Pacific* factor 15 is a bargaining situation, that is, a game situation whose outcome depends on the players' bargaining strategies. Such a bargaining situation can be modeled as a two-player nonzero sum game, where the gain of one player is the loss of the other.[11] In this regard, game-theoretic methodology is currently the best available tool for formalizing the bargaining process and outcome.[12] Thus, part II describes various game-theoretical bargaining solutions to model the hypothetical negotiation of *Georgia-Pacific* factor 15.

Georgia-Pacific factor 15 is setting forth the framework through which the factors, 1 through 14, are applied.[13]

1986) (citing Panduit Corp. v. Stahlin Bros. Fibre Works, Inc., 575 F.2d 1152, 1157–58 (6th Cir. 1978)); 7 DONALD S. CHISUM, CHISUM ON PATENTS § 20.07[2][d] (2011).

[9] *Georgia-Pacific*, 318 F. Supp. at 1120. In the following, factor *x* of *Georgia-Pacific* will be referred to as *Georgia-Pacific* factor *x* or simply factor *x*.

[10] *Id.*

[11] ABHINAY MUTHOO, BARGAINING THEORY WITH APPLICATIONS 6 (1999).

[12] *Id.*; RICHARD F. CAULEY, WINNING THE PATENT DAMAGES CASE: A LITIGATOR'S GUIDE TO ECONOMIC MODELS AND OTHER DAMAGE STRATEGIES 28–29 (2009) ("Game theory may provide a solution, or at least an approach, to solving this problem.").

[13] Mobile Oil Corp. v. Amoco Chems. Corp., 915 F. Supp. 1333, 1367–68 (D. Del. 1994); RUSSELL L. PARR & GORDON V. SMITH, INTELLECTUAL PROPERTY: VALUATION, EXPLOITATION, AND INFRINGEMENT DAMAGES 172 (4th ed. 2005 & Supp. 2011); John J. Barnhardt, III, *Revisiting a Reasonable Royalty as a Measure of Damages for Patent Infringement*, 86 J. PAT. & TRADEMARK OFF. SOC'Y 991, 992 (2004); Daralyn J. Durie & Mark A. Lemley, *A Structured Approach to Calculating Reasonable Royalties*, 14 LEWIS & CLARK L. REV. 627,

Accordingly, in part III *Georgia-Pacific* factors 1 through 14 are correlated to individual elements of the game-theoretical bargaining solution. "A court or jury may look at any or all of [these fourteen] factors [as far as they] influence the [particular] hypothetical negotiation [thereby] assign[ing] appropriate weight to the factors on a case-by-case basis."[14] "[N]o[t] one [of them] is dispositive, and indeed, not all of [them] will be relevant in each case."[15]

II. FRAMEWORK OF THE HYPOTHETICAL NEGOTIATION

Game-theoretic bargaining solutions "can be divided in two [approaches]: the cooperative and the noncooperative approach."[16] The cooperative approach is axiomatic, that is, axioms are formulated and if they are satisfied, a unique bargaining outcome exists.[17] On the other hand, the noncooperative approach simulates the strategic behavior of the players, that is, the negotiation process.[18] The hypothetical bargaining under *Georgia-Pacific* factor 15 can be modeled both as a cooperative as well as a noncooperative bargaining approach.[19]

In the following, first the Nash bargaining solution and other basic cooperative bargaining solutions are described. Thereafter, the Rubinstein bargaining solution, a basic noncooperative bargaining solution, is addressed. Consistent with the hypothetical approach of *Georgia-Pacific*, both bargaining solution approaches assume complete information, that is, the bargainers' utility functions, the set of feasible agreements, and the outside options are common knowledge. Any uncertainty

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643 (2010).

[14] CHRISTOPHER M. ARENA & EDUARDO M. CARRERAS, THE BUSINESS OF INTELLECTUAL PROPERTY 345 (2008). *See* Amy L. Landers, *Let the Games Begin: Incentives to Innovation in the New Economy of Intellectual Property Law*, 46 SANTA CLARA L. REV. 307, 325–27 (2006) (noting that courts may consider factors such as "prior license rates obtained by the patentee, the commercial relationship between the patentee and the infringer, and the market rate for the patented invention that the parties would have reached if a rate had been negotiated between them." (citation omitted)).

[15] ARENA & CARRERAS, *supra* note 14, at 345.

[16] HANS PETERS, GAME THEORY: A MULTI-LEVELED APPROACH 297 (2008).

[17] Bruno Verbeek & Christopher Morris, *Game Theory and Ethics,* STANFORD ENCYCLOPEDIA OF PHILOSOPHY (Oct. 16, 2004, revised June 8, 2010), http://www.illc.uva.nl/~seop/entries/game-ethics.

[18] *Id.*

[19] *See infra* Part II.C.

present in the situation would be shared.[20]

A. Cooperative Bargaining Solutions

The basic cooperative bargaining solution was introduced by John Nash.[21] Based on Nash's work, other bargaining solutions, such as the Kalai-Smorodinsky and the egalitarian bargaining solution, were developed.[22]

1. Nash Bargaining Solution

> Nash defin[ed] a "bargaining problem" to be the set of utility pairs that can be derived from possible agreements, together with a [utility] pair . . . [that represents] the "disagreement point". A function that assigns a single outcome to every such problem is a "bargaining solution". Nash propose[d] that a bargaining solution should satisfy four [axioms]. [And showed] . . . that there is only one solution that does so, which is known as the Nash bargaining solution.[23]

The axioms of the Nash bargaining solution are:

Pareto Optimality: $f(S)$ is a Pareto optimal coordinate point in a solution set S, that is, there is no point $p \epsilon S$ with $p > f(S)$.[24] In other words, it is not possible to increase the utility of one player without decreasing, at the same, time the utility of the other player.[25]

Symmetry: If the set S is symmetric, then $f(S) = (u, u)$.[26] This means that the bargaining solution stays the same if the players change roles.[27]

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[20] *See* Kalyan Chatterjee, *Disagreement in Bargaining: Models with Incomplete Information*, *in* GAME-THEORETIC MODELS OF BARGAINING 9, 10 (Alvin E. Roth ed., 1985).

[21] *See* Tanja Magoc & Vladik Kreinovich, *A New Simplified Derivation of Nash Bargaining Solution*, 3 APPLIED MATHEMATICAL SCI. 1097 (2009) (explaining Nash's work).

[22] Bram Driesen, Andrés Perea & Hans Peters, *The Kalai-Smorodinsky Bargaining Solution with Loss Aversion*, 61 MATHEMATICAL SOC. SCI. 58, 58 (2011).

[23] MARTIN J. OSBORNE & ARIEL RUBINSTEIN, BARGAINING AND MARKETS 3 (Karl Shell ed., 1990).

[24] *See id.* at 13 (it follows that at an inferior outcome will result in renegotiation until the Pareto Optimal point is reached); Kamal Jain & Mohammad Mahdian, *Cost Sharing*, *in* ALGORITHMIC GAME THEORY 385, 405 (Noam Nisan et al. eds., 2007).

[25] *See* OSBORNE & RUBINSTEIN, *supra* note 23, at 13.

[26] *See id.* at 12; Jain & Mahdian, *supra* note 24, at 405.

[27] OSBORNE & RUBINSTEIN, *supra* note 23, at 12.

Scale Independence: The solution is independent of the scale used to measure individual utilities, that is, if S' is obtained from S by multiplying all utilities of the ith player by λ_i, then $f(S')$ can be obtained from $f(S)$ by multiplying the ith coordinate by λ_i.[28]

Independence of Irrelevant Alternatives: If $S'' \subset S$ and $f(S) \in S''$, then $f(S'') = f(S)$.[29] This means that if a proper subset of bargaining alternatives, S'', is considered, a solution to S contained in S'' is also a solution to S''.[30]

From the nature of the Nash bargaining solution as an axiomatic bargaining approach, it follows that "there is a unique solution for [a] bargaining [problem] satisfying Pareto optimality, symmetry, scale independence, and independence of irrelevant alternatives."[31] Assuming a bargaining of two players, Nash showed that the only solution that satisfies all axioms is the one obtained by solving the constrained maximization problem

$$\max \Omega = (\pi_1 - d_1)(\pi_2 - d_2), \qquad (1)$$

where π_1 and π_2 denote player 1's and 2's payoff from licensing, respectively, and d_1 and d_2 denote their respective disagreement payoffs if they would pursue outside options.[32] The solution of the Nash bargaining is illustrated in figure 1, where f is the function to be maximized, S represents the solution set, m_1 and m_2 denote the maximum payoffs of player 1 and 2, respectively, and d_1 and d_2 denote their respective disagreement payoffs.

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[28] *Id.* at 11–12; Jain & Mahdian, *supra* note 24, at 405.

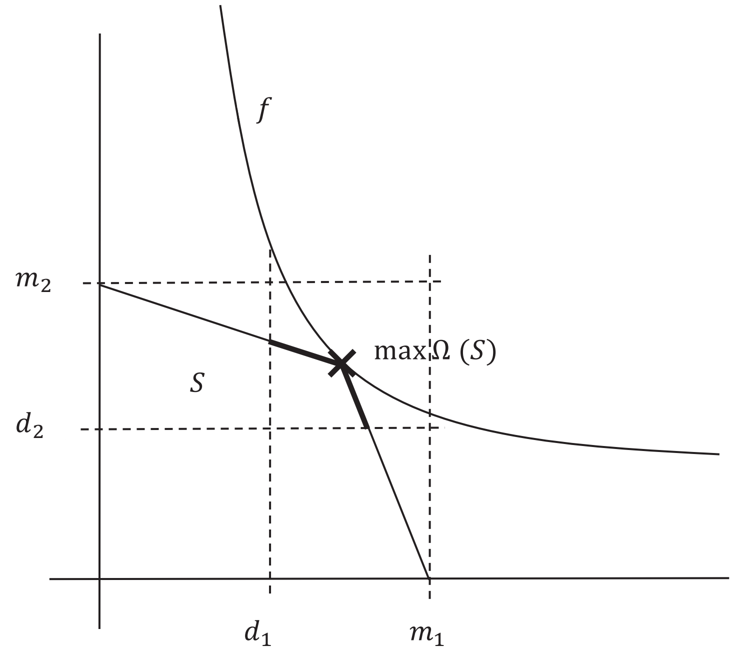
[29] Jain & Mahdian, *supra* note 24, at 405.

[30] *See id.* at 405 (noting that the solution is independent irrelevant of alternatives); William Choi & Roy Weinstein, *An Analytical Solution to Reasonable Royalty Rate Calculations*, 41 IDEA 49, 53 (2001) (explaining the Nash bargaining solution).

[31] Jain & Mahdian, *supra* note 24, at 405. *See also* John Nash, *Two-Person Cooperative Games*, 21 ECONOMETRICA 128, 129 (1953) ("One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely.").

[32] Choi & Weinstein, *supra* note 30, at 54.

Figure 1



All points in S are possible outcomes of the bargaining. However, because neither player will enter into an agreement resulting in a payoff worse than its disagreement payoff, the solution must be located above the horizontal line through d_2 and to the right of the vertical line through d_1.[33] Further, the solution must be located on the border of S (denoted in bold) because other points in the set are not Pareto optimal.[34] Now, it can be observed that the solution is found at the corner of the Pareto frontier because this point represents the maximum value for $\max \Omega = (\pi_1 - d_1)(\pi_2 - d_2)$.[35]

This solution implies that

$$(\pi_1 - d_1) = (\pi_2 - \pi_2) \qquad (2)$$

because $\max \Omega = (\pi_1 - d_1)\,(\pi_2 - d_2)$ will only evaluate to a

[33] *See id.* (explaining that the disagreement payoff is the amount either party would receive if the negotiation falls through).

[34] *See id.* at 53–54 ("[T]here should be no other feasible allocation that is better than the solution for one negotiator and not worse than the solution for the other negotiator.").

[35] William Choi and Roy Weinstein provide an example of this. Choi & Weinstein, *supra* note 30, at 54–55.

maximum value if $(\pi_1 - d_1)$ and $(\pi_2 - d_2)$ are equal.[36]

For obtaining player 1's and 2's payoff according to the Nash bargaining solution, let Π denote the total expected payoff if the negotiations are successful.[37] Now, if the players decide to bargain over partitioning the total expected payoff, they first agree to give each other their respective disagreement payoffs and then they split the remaining payoff equally.[38] Thus, if the negotiations are successful, the payoff for player 1 will be

$$\pi_1 = d_1 + \frac{1}{2}(\Pi - d_1 - d_2), \qquad (3)$$

and the payoff for player 2 will be

$$\pi_2 = d_2 + \frac{1}{2}(\Pi - d_1 - d_2).^{39} \qquad (4)$$

It can be observed that there will be no successful bargaining if the total expected payoff is smaller than the sum of the disagreement payoffs. In such case, at least one of the players can gain a higher disagreement payoff than the payoff it could gain from the bargaining.[40] Thus, no negotiations would take place and both players would pursue their outside options realizing their respective disagreement payoffs.[41]

2. Nash Bargaining Solution with Unequal Bargaining Power

The Nash bargaining solution can be extended to account for unequal bargaining power of the players.[42] Denoting the bargaining power of player 1 relative to player 2 by α, where $0 \leq \alpha \leq 1$, player 1 receives a payoff of

$$\pi_1 = d_1 + \alpha(\Pi - d_1 - d_2), \qquad (5)$$

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[36] *See id.* at 55.
[37] *Id.* at 54.
[38] *Id.* at 55.
[39] *See id.*
[40] *Id.* at 54–55.
[41] *See* Mark A. Glick, *The Law and Economics of Patent Infringement Damages*, UTAH B.J., Mar. 1997, at 11, 15 ("[I]t must be assumed that there are mutual gains from voluntary trade, that is, that both the licensor and the licensee will be better off . . . .").
[42] *See* John C. Jarosz & Michael J. Chapman, *Application of Game Theory to Intellectual Property Royalty Negotiations*, *in* LICENSING BEST PRACTICES: STRATEGIC, TERRITORIAL, AND TECHNOLOGY ISSUES 241, 252 (Robert Goldscheider & Alan H. Gordon eds., 2006) (stating that the equation is dependent on bargaining strength).

and player 2 receives a payoff of

$$\pi_2 = d_2 + (1 - \alpha)(\Pi - d_1 - d_2).^{43} \tag{6}$$

3. Kalai-Smorodinsky Bargaining Solution

The bargaining solution introduced by Ehud Kalai and Meir Smorodinsky is based on the Nash bargaining solution, however, replaces the axiom of independence from irrelevant alternatives with an axiom of monotonicity.[44] Kalai and Smorodinsky explain the replacement using an example of two solution sets

$$\begin{gathered} S_1 = convex\ hull\ \{(0,1),(1,0),(0.75,0.75)\}, \text{and} \\ S_2 = convex\ hull\ \{(0,1),(1,0),(1,0.7)\}.^{45} \end{gathered} \tag{7}$$

They further assume for their example that both players' disagreement payoffs are zero, that is, it holds that

$$d_1 = d_2 = 0.^{46} \tag{8}$$

Thus, maximizing the payoff according to the Nash bargaining solution, that is, applying $\max \Omega = (\pi_1 - d_1)(\pi_2 - d_2)$ to S_1, leads to a maximum payoff of

$$\max \Omega(S_1) = (0.75 - 0)(0.75 - 0) = 0.5625.^{47} \tag{9}$$

For S_2, however, the maximum payoff is

$$\max \Omega(S_2) = (1 - 0)(0.7 - 0) = 0.7.^{48} \tag{10}$$

Therefore, if $\max \Omega\,(S_1)$ is the initial solution to a bargaining problem and the solution set is later extended to S_2, as shown in figure 2,[49] the solution changes to $\max \Omega\,(S_2)$.[50]

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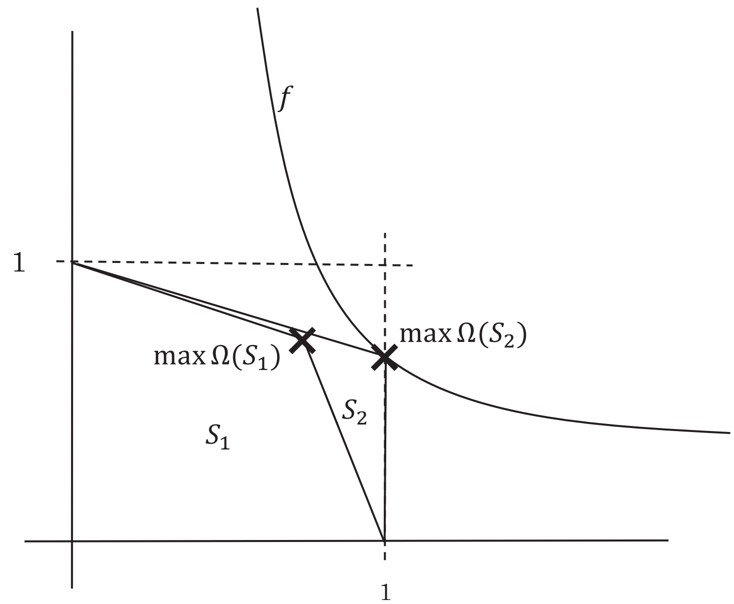
[43] *Id.* at 248.
[44] Ehud Kalai & Meir Smorodinsky, *Other Solutions to Nash's Bargaining Problem*, 43 ECONOMETRICA 513, 514–15 (1975).
[45] *Id.* at 515.
[46] *See id.*
[47] Substituting the solution values from the solution set S_1.
[48] Substituting the solution values from the solution set S_2.
[49] *Compare* Kalai & Smorodinsky, *supra* note 44, at 515, *with infra* figure 2.
[50] *See* Kalai & Smorodinsky, *supra* note 44, at 515.

Figure 2



In such case, player 1's payoff is increased from 0.75 to 1, while player 2's payoff is decreased from 0.75 to 0.7.[51] This result of the Nash bargaining solution can be criticized because the payoff for player 2 is decreased even though the solution set has extended for *both* players.

In order to avoid the result of the Nash bargaining solution, Kalai and Smorodinsky replace the axiom of independence from irrelevant alternatives with the axiom of monotonicity.[52] The axiom of monotonicity states, "if, for every utility level that player 1 may demand, the maximum feasible utility level that player 2 can simultaneously reach is increased, then the utility level assigned to player 2 according to the solution should also be increased."[53]

Applying the axiom of monotonicity to the solution set S_2, player 2 is unable to increase its payoff beyond player 1's

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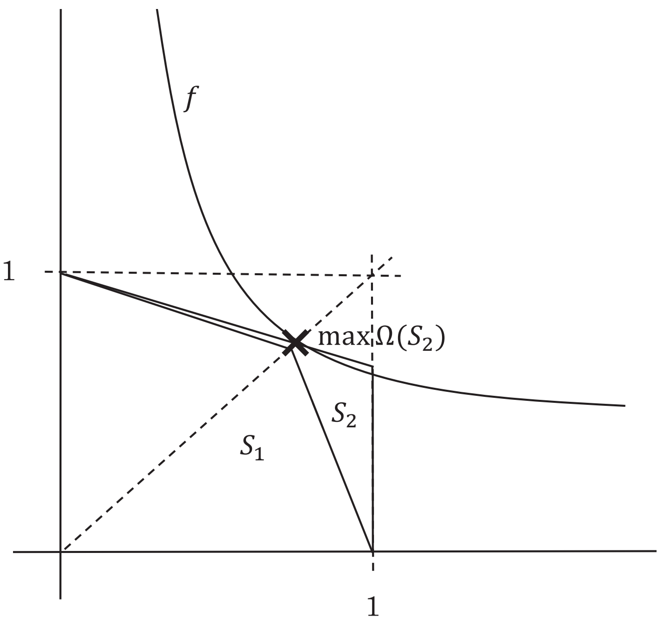
[51] *See id.* (comparing S_1's solution, (0.75, 0.75), to S_2's solution, (1, 0.7), one can infer that player 1's payoff, plotted on the *x*-axis, increased from 0.75 to 1 and player 2's payoff, plotted on the *y*-axis, decreased from 0.75 to 0.7).

[52] *Id.* at 513–516.

[53] *Id.* at 515.

increase.[54] To obtain the solution, since the disagreement payoffs are zero, a line can be drawn through points (0, 0) and (1, 1), in which case the solution will be determined by the point where the line crosses the border of the solution set S_2, as can be observed in figure 3.[55]

Figure 3



Thus, the maximum payoff under the Kalai-Smorodinsky solution for the solution set S_2 is

$$\max \Omega(S_2) \approx (0.77 - 0)(0.77 - 0) = 0.5929.^{56} \qquad (11)$$

4. Egalitarian Bargaining Solution

Ehud Kalai's egalitarian bargaining solution is based on John Rawls Max-Min approach to the theory of justice.[57] According to

[54] *Id.* at 515–17.
[55] *See id.* at 515–18.
[56] *See id.*
[57] *See* Ehud Kalai, *Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons*, 45 ECONOMETRICA 1623, 1629–30 (1977) (describing the applicability of Kalai's presented bargaining theory to Rawls'

this approach, social and economic resources are to be arranged so that they are of greatest benefit to the least-advantaged.[58] Similarly, under the egalitarian bargaining solution, players will maximize their utilities subject to the restriction that they all gain equally.[59] The egalitarian bargaining solution is a modification of the Nash and Kalai-Smorodinsky bargaining solutions.[60] It is Pareto optimal, symmetric, independent of irrelevant alternatives, and monotone.[61] However, in addition it is also translation independent.[62] Thus, for example, if two players are given one hundred chips, each of which is worth one dollar, the players would share the chips equally.[63] However, if player 1 could cash in one chip for three dollars, while player 2 still could only cash in each chip for one dollar, the division would be 25 chips for player 1 and 75 chips for player 2.[64]

B. Noncooperative Bargaining Solutions

In contrast to the cooperative bargaining solutions, such as the Nash, Kalai-Smorodinsky, or egalitarian bargaining solution, the noncooperative bargaining approach obtains its result by modeling the bargaining process.

1. Rubinstein Bargaining Solution

The noncooperative bargaining approach was promoted by Ariel Rubinstein, who developed a bargaining solution with two players making alternating offers and counteroffers, as shown in figure 4,[65] which ultimately converge towards an equilibrium.[66]

Max-Min approach).

[58] *See generally* JOHN RAWLS, A THEORY OF JUSTICE (Harvard Univ. Press rev. ed. 1999).

[59] Kalai, *supra* note 57, at 1623.

[60] *See id.* at 1623–25, 1629–30.

[61] Shiran Rachmilevitch, *Disagreement Point Axioms and the Egalitarian Bargaining Solution*, 40 INT'L J. GAME THEORY 63, 65 (2011).

[62] *Id.*

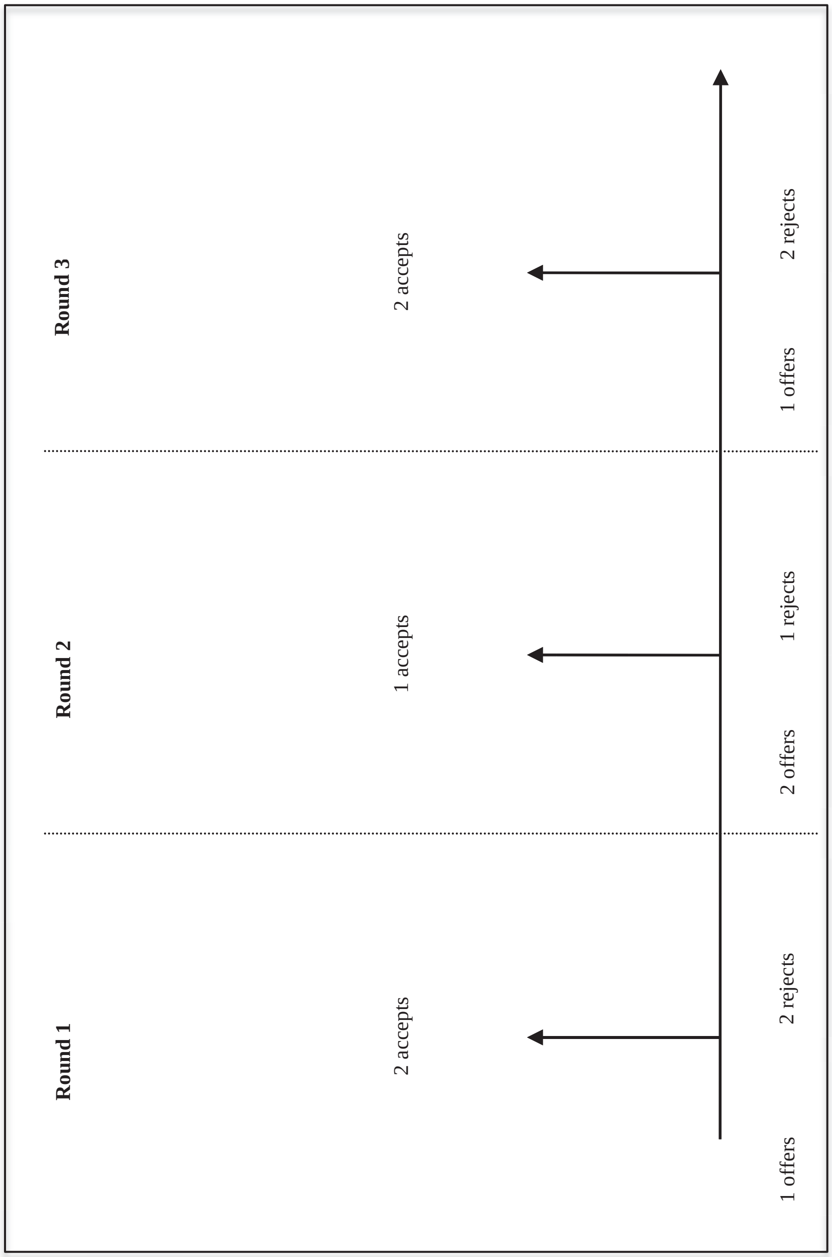
[63] Kalai, *supra* note 57, at 1629.

[64] *Id.*

[65] *See* JÜRGEN JERGER, SPIELTHEORIE 120 fig. 6.3 (2006) (figure 4 is modified from Jerger's work).

[66] Ariel Rubinstein, *Perfect Equilibrium in a Bargaining Model*, 50 ECONOMETRICA 97, 97 (1982). Rubinstein's bargaining solution is based on the work of Ingolf Ståhl. *See* OSBORNE & RUBINSTEIN, *supra* note 23, at 65 ("The first to investigate the alternating offer procedure was Ståhl . . . .").

Figure 4


Round 1
Round 2
Round 3
2 accepts
1 accepts
2 accepts
1 offers
2 rejects
2 offers
1 rejects
1 offers
2 rejects

The Rubinstein bargaining solution uses the notion of a subgame perfect equilibrium.[67] A subgame perfect equilibrium exists if the players' strategies converge towards a Nash equilibrium (i.e., no player can achieve a higher payoff by only changing its own strategy) in every subgame.[68] This can lead to a single solution for the game, the perfect equilibrium partition.[69] Considering a bargaining in which the payoff is discounted each round, the perfect equilibrium partition results from

$$\overline{s} = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \tag{12}$$

where $\overline{s}$ is the maximum share of player 1, δ_1 and δ_2 are the discount factors of players 1 and 2, respectively, and the total expected payoff is normalized to 1.[70] The unique perfect equilibrium partition can be obtained by backward induction.[71]

As shown in table 1, starting with round 3, it is assumed that player 1 can claim a maximum share of $\overline{s}$. Thus, in round 2, when it is player 2's turn, player 2 knows that it must offer player 1 a share of $\delta_1 \overline{s}$, thus, can decrease player 1's share by its discount factor.[72] Player 1 would accept the offer because receiving $\delta_1 \overline{s}$ in round 2 is equal to receiving $\overline{s}$ in round 3.[73] The shares are equal from the perspective of player 1 because receiving $\delta_1 \overline{s}$ in round 2 has the same value as receiving $\overline{s}$ in round 3.[74] Given that the total expected payoff is normalized to 1, player 2's share in round 2 will be $1 - \delta_1 \overline{s}$.

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67 Rubinstein, *Perfect Equilibrium*, *supra* note 66, at 97, 99, 102. The subgame perfect equilibrium was introduced by Reinhard Selten. *The Concise Encyclopedia of Economics: Reinhard Selten*, LIBR. ECON. & LIBERTY, http://www.econlib.org/library/Enc/bios/Selten.html (last visited Mar. 2, 2012).

68 OSBORNE & RUBINSTEIN, *supra* note 23, at 44.

69 Rubinstein, *Perfect Equilibrium*, *supra* note 66, at 99.

70 *Id.* at 99, 108.

71 OSBORNE & RUBINSTEIN, *supra* note 23, at 54.

72 JERGER, *supra* note 65, at 120.

73 *Id.*

74 *Id.*

Table 1

Round	Offering Player	Player 1's Share	Player 2's Share
3	1	$\overline{s}$	
2	2	$\delta_1\overline{s}$	$1-\delta_1\overline{s}$
1	1	$1-\delta_2(1-\delta_1\overline{s})$	$\delta_2(1-\delta_1\overline{s})$

Proceeding backwards to round 1, it is again player 1's turn to make an offer. In order to ensure that player 2 will accept its offer, player 1 must propose the amount player 2 could claim in the second round, taking into account player 2's discount factor.[75] Thus, player 1 would offer player 2 a share of $\delta_2(1-\delta_1\overline{s})$ and would keep a share of $1-\delta_2(1-\delta_1\overline{s})$.[76] Now, at this point it can be observed that for player 1 receiving a share of $\overline{s}$ in round 3 is the same as receiving a share of $1-\delta_2(1-\delta_1\overline{s})$ in round 1.[77] Therefore, it can be concluded that

$$\overline{s} = 1-\delta_2(1-\delta_1\overline{s}).^{78} \tag{13}$$

Solving this equation for $\overline{s}$, player 1's share is expressed by equation (12), which is the perfect equilibrium partition.

The perfect equilibrium partition is unique, which can be shown by backward induction on $\underline{s}$, the minimum share that player 1 can obtain. Using a similar argument as for the maximum share the result is

$$\underline{s} = \frac{1-\delta_2}{1-\delta_1\delta_2}. \tag{14}$$

Observing that $\overline{s} = \underline{s}$, it can be concluded that the perfect equilibrium partition is unique.

2. Rubinstein Bargaining Solution with Disagreement Payoffs

The Rubinstein bargaining solution can be extended to account for disagreement payoffs, d_1 and d_2.[79] Similar as in the

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[75] *Id.*
[76] *Id.*
[77] *Id.*
[78] *Id.*
[79] *See id.* at 122.

Rubinstein bargaining solution without disagreement payoffs, players can either accept an offer or reject an offer by making a counteroffer.[80] However, in the Rubinstein bargaining solution with disagreement payoffs they can also terminate the bargaining and realize their disagreement payoffs.[81] If a player has no outside option, its disagreement payoff would be zero.[82] Further, the sum of the disagreement payoffs cannot be greater than the total expected payoff from the bargaining because otherwise at least one of the players would have no interest in the bargaining in the first place.[83] The backward induction for the Rubinstein bargaining solution with disagreement payoffs is shown in table 2.[84]

Table 2

Round	Offering Player	Player 1's Share	Player 2's Share
3	1	s	
2	2	$\max(\delta_1 s, d_1)$	$1 - \max(\delta_1 s, d_1) \geq d_2$
1	1	$1 - \max[\delta_2(1 - \delta_1 s), d_2] \geq d_1$	$\max[\delta_2(1 - \delta_1 s), d_2]$

80 *Id.*

81 *Id.*

82 *See id.*; Choi & Weinstein, *supra* note 30, at 54 (explaining that the disagreement payoff is the amount either party would receive if the negotiation falls through).

83 *See supra* Part II.A.1 (describing that a player will not accept a payoff that is lower in value than the disagreement payoff received).

84 Jerger provides an explanation of the following backward induction. *See* JERGER, *supra* note 65, 122–24.

Generally, if in any round of the bargaining, one player does not offer the other player a share at least as large as its disagreement payoff, the bargaining will be terminated.[85] Otherwise, the solution follows a similar reasoning as the solution without disagreement payoffs.[86] Thus, for player 1, receiving a share of s in round 3 is of equal value as receiving a share of $1 - \max[\delta_2(1 - \delta_1 s), d_2]$ in round 1. Therefore,

$$\begin{aligned} s &= 1 - \max[\delta_2(1 - \delta_1 s), d_2] \\ \mathrm{s} &= \min[1 - \delta_2(1 - \delta_1 s), 1 - d_2]. \end{aligned} \tag{15}$$

Solving for s using the results from the Rubinstein bargaining solution without disagreement payoffs,[87] the share of player 1 is obtained by

$$s = \min\left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, 1 - d_2\right), \tag{16}$$

which is the unique perfect equilibrium partition.[88]

C. Relationship Between Cooperative & Noncooperative Bargaining

The cooperative and noncooperative approaches to the bargaining problem "justify and clarify [each] other."[89] Particularly, depending on the definition of the problem, the Rubinstein bargaining solution can lead to results "close to the Nash [bargaining] solution."[90] Thus, it can be shown that the outcome of the Rubinstein bargaining solution converges to the outcome of the Nash bargaining solution.[91] Therefore, the hypothetical negotiation under *Georgia-Pacific* factor 15 can be based on both the Nash and Rubinstein bargaining solution. Furthermore, it can be based on other cooperative or

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[85] *See id.* at 122–23.
[86] *See id.* (detailing the outcome of negotiations without outside options and disagreement payoffs).
[87] *See supra* Part II.B.1 (discussing the Rubinstein bargaining solution).
[88] *See id.*
[89] Nash, *supra* note 31, at 129; OSBORNE & RUBINSTEIN, *supra* note 23, at 70 (discussing the Nash bargaining solution).
[90] OSBORNE & RUBINSTEIN, *supra* note 23, at 3.
[91] *See* HAROLD HOUBA & WILKO BOLT, CREDIBLE THREATS IN NEGOTIATIONS: A GAME-THEORETIC APPROACH 107 (2002) (discussing the similar outcomes of the Nash and Rubinstein bargaining solutions). *See generally* Ken Binmore et al., *The Nash Bargaining Solution in Economic Modelling*, 17 RAND J. ECON. 176 (1986) (comparing the Rubinstein and Nash bargaining solutions).

noncooperative bargaining solutions as well.

III. DETAILS OF THE HYPOTHETICAL NEGOTIATION

If the patentee already received royalties for licensing the patent in the past, those royalties may be conclusive for the outcome of the hypothetical negotiation.[92] Accordingly, *Georgia-Pacific* factor 1 permits consideration of "[t]he royalties received by the patentee for the licensing of the patent in suit, proving or tending to prove an established royalty."[93] To show that "a royalty [is] established, it [first] 'must be paid by such a number of [licensees] as to indicate a general acquiescence in its reasonableness by those who have occasion to use the invention.'"[94] Also, "it must be paid or secured before the infringement complained of . . . and it must be uniform at the places where the licenses are issued."[95] Further, settlement licenses, that is, licenses that were negotiated as part of an agreement to resolve a patent infringement dispute, are generally not considered.[96] If an established royalty satisfies these requirements, it may be the best measure of damages.[97]

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[92] *See, e.g.*, Uniloc USA, Inc. v. Microsoft Corp., 632 F.3d 1292, 1317–18 (Fed. Cir. 2011) (stating that looking at royalties paid or received for the purpose of a hypothetical negotiation may be allowed).

[93] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F.Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

[94] Trell v. Marlee Elecs. Corp., 912 F.2d 1443, 1446 (Fed. Cir. 1990) (citing Hanson v. Alpine Valley Ski Area, Inc., 718 F.2d 1075, 1078 (Fed. Cir. 1983)).

[95] Rude v. Westcott, 130 U.S. 152, 165 (1889).

[96] Faulkner v. Gibbs, 199 F.2d 635, 638 (9th Cir. 1952); Universal Athletic Sales Co. v. Am. Gym, Recreational & Athletic Equip. Corp., 480 F. Supp. 408, 414 (W.D. Pa. 1979); Glick, *supra* note 41, at 15 n.15. *But see* Michael J. Chapman, *Using Settlement Licenses in Reasonable Royalty Determinations*, 49 IDEA 313, 356 (2009) ("[S]ettlement licenses can be – and should be – considered along with all other available evidence in reasonable royalty determinations.").

[97] Monsanto Co. v. McFarling, 488 F.3d 973, 978–79 (Fed. Cir. 2007) ("An established royalty is usually the best measure of a 'reasonable' royalty for a given use of an invention . . . ."); *Faulkner*, 199 F.2d at 638 (stating that an established royalty is "the best measure of the value of what was taken by the infringement," and listing the requirements necessary to qualify as an "established" royalty); Tektronix, Inc. v. United States, 552 F.2d 343, 347 (Ct. Cl. 1977) ("Where an established royalty rate for the patented inventions is shown to exist, that rate will usually be adopted as the best measure of reasonable and entire compensation."); Mobil Oil Corp. v. Amoco Chems. Corp., 915 F. Supp. 1333, 1353 (D. Del. 1994) ("[T]he royalties received by the patentee for the licensing of the patents in suit is the 'most influential factor' in determining a reasonable royalty."); Chapman, *supra* note 96, at 323 (stating that the established royalty is "the best measure of the reasonable royalty that should be paid by an infringer").

If an established royalty does not exist, however, the hypothetical negotiation has to be further evaluated. Accordingly, in the following the details of the hypothetical negotiation are modeled based on the Nash bargaining solution.[98] As described earlier, the Nash bargaining solution with unequal bargaining power states that the payoff of player 1 (in the following, the patentee)[99] is determined by the payoff function

$$\pi_1 = d_1 + \alpha(\Pi - d_1 - d_2), \tag{17}$$

and the payoff of player 2 (in the following, the infringer) is determined by the payoff function

$$\pi_2 = d_2 + (1 - \alpha)(\Pi - d_1 - d_2).^{100} \tag{18}$$

From these functions it can be observed that the respective payoffs of the patentee and infringer, π_1 and π_2, depend on three values: the total expected payoff, Π, the respective disagreement payoffs, d_1 and d_2, and the respective bargaining powers, α and $1 - \alpha$.[101] In the following, it will be addressed how these values can be computed for purposes of a reasonable royalty analysis and how they relate to *Georgia-Pacific* factors 1–14.

A. Total Expected Payoff

Payoff, in general, is a player's valuation of the outcome of a game, such as profits or cost savings.[102] Calculating a reasonable royalty depends on the division of the infringer's profit between the patentee and the infringer.[103] Thus, for the purpose of reasonable royalty determination, the total expected payoff can relate to profit, more specifically, to the profit that is attributable to commercialization of the infringed intellectual property right.[104] However, since cost savings result in higher profit, the

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[98] As described in Part II.C., the Nash and Rubinstein bargaining solutions converge to the same equilibrium, thus, it would be possible as well to use the Rubinstein bargaining solution.

[99] In the following "the word 'patentee' includes not only the patentee to whom the patent was issued but also the successors in title to the patentee." 35 U.S.C.A. § 100(d) (West, Westlaw through P.L. 112-89 (excluding P.L. 112-55, 112-74, 112-78, and 112-81) approved 1/3/12).

[100] *See supra* Part II.A.2.

[101] *See supra* Part II.A.1–2.

[102] JEFFREY M. PERLOFF, MICROECONOMICS: THEORY & APPLICATIONS WITH CALCULUS 492 (int'l ed. 2008).

[103] *See* Glick, *supra* note 41, at 15 (examining the different aspects of valuation in relation to a patentee and licensee).

[104] *See id.*

total expected payoff can also refer to cost savings.[105] For example, using a patented process can decrease manufacturing costs thereby increasing profit.[106] In this sense, cost savings are a complementary way of looking at increase of profit. While in the following it is assumed that the infringement occurs due to selling an infringing product, a reasonable royalty determination for infringing processes would be very similar.

1. Anticipated Profit

In order to calculate profit, courts give considerable weight to the infringer's anticipated profit.[107] The theory is that the patentee and the infringer would have negotiated a royalty based on the anticipated profit that the infringer could realize by adopting the patentee's invention.[108] In this regard, it is assumed that the infringer wants to maximize profit whenever possible, which is not necessarily true in reality.[109]

Generally, profit, Π, is defined as a firm's revenue, r, minus its cost, c, both dependent on the quantity, q, the firm produces.[110] Formally this is expressed by

$$\Pi(q) = r(q) - c(q), \tag{19}$$

[105] Tilghman v. Proctor, 125 U.S. 136, 146 (1888) ("[I]f the [infringer] gained an advantage by using the [patentee's] invention, that advantage is the measure of the profits to be accounted for . . . ."); Landers, *supra* note 14, at 314–15 ("'[P]rofits' . . . include[ ] any advantage that the defendant gained by the patent, such as a defendant's manufacturing cost savings if attributable to the patented invention."); Mark A. Lemley & Carl Shapiro, *Patent Holdup and Royalty Stacking*, 85 TEX. L. REV. 1991, 1996 (2007) (discussing the payoff in the context of valuation and reasonable royalty).

[106] *See* Eric E. Bensen & Danielle M. White, *Using Apportionment to Rein in the Georgia-Pacific Factors*, 9 COLUM. SCI. & TECH. L. REV. 1, 49 (2008) ("If the patent is for a process or for a device used in a product to reduce manufacturing costs, the cost savings enjoyed from practicing the patent would be subject to the royalty.").

[107] *See* Trell v. Marlee Elecs. Corp., 912 F.2d 1443, 1446 (Fed. Cir. 1990) ("[T]he . . . court may consider the infringer's anticipated profits, as indicated by . . . actual profits [when determining royalties]."); Trans-World Mfg. Corp. v. Al Nyman & Sons, Inc., 750 F.2d 1552, 1568 (Fed. Cir. 1984) (explaining that anticipated profits may be considered when determining royalties).

[108] CHISUM, *supra* note 8, § 20.07[2][d].

[109] ROBERT S. PINDYCK & DANIEL L. RUBINFELD, MICROECONOMICS 274 (7th ed. 2009).

[110] *Id.* at 276.

while revenue is equal to price, p, times quantity, that is,

$$r(q) = pq.^{111} \tag{20}$$

With regard to the cost, it should be noted that it does not include the royalties to be paid by the infringer to the patentee because this amount is what is to be determined by modeling the hypothetical negotiation in the first place.[112] However, if the infringer licensed other intellectual property beyond the infringed right, these licensing costs are included in the cost.[113]

The amount of profit the infringer can anticipate depends on the market in which a product will be sold.[114] Thus, the hypothetical negotiation necessarily involves reconstruction of the relevant market.[115] Consideration of the market structure is reflected in *Georgia-Pacific* factor 3, which refers to "[t]he nature and scope of the license, as exclusive or nonexclusive; or as restricted or nonrestricted in terms of territory or with respect to whom the manufactured product may be sold."[116] Thus, for example, if the infringer expected to be the single seller in the market of the licensed invention, the license would have been exclusive and the market would be a monopoly.[117]

Based on the foregoing, calculating the anticipated profit begins with a determination of the market in which the infringer expected to sell the product.[118] As implied by *Georgia-Pacific* factor 3, the classification of a market can be based on the number of firms in the market that supply a product and on the ability of customers to distinguish between the products of the

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[111] *Id.* In the following it will no longer be explicitly mentioned that revenue and costs, and consequently profit, are dependent on quantity.

[112] CHISUM, *supra* note 8, § 20.07[2][d].

[113] *See id.*

[114] *See* Uniloc USA, Inc. v. Microsoft Corp., 632 F.3d 1292, 1317 (Fed. Cir. 2011) (quoting ResQnet.com, Inc. v. Lansa, Inc., 594 F.3d 860, 869 (Fed. Cir. 2010) ("[E]xpert testimony opining on a reasonable royalty rate must 'carefully tie proof of damages to the claimed invention's footprint in the market place.'"); Grain Processing Corp. v. Am. Maize-Prods. Co., 185 F.3d 1341, 1350 (Fed. Cir. 1999) (requiring a "reconstruction of the market").

[115] Riles v. Shell Exploration & Prod. Co., 298 F.3d 1302, 1311 (Fed. Cir. 2002); *id.*; Vincent E. O'Brien, *Economics and Key Patent Damages Cases*, 9 U. BALT. INTELL. PROP. L.J. 1, 16 (2000) (citations omitted). *See also* LAWRENCE M. SUNG, PATENT INFRINGEMENT REMEDIES 305–08 (2004) (providing extended examples of market reconstruction theory applied to other case law).

[116] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1119-20 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

[117] *See infra* Part III.A.1.b.

[118] *Grain Processing*, 185 F.3d at 1350.

different firms.[119] In the following, four basic market models will be described: perfect competition, monopoly, monopolistic competition, and oligopoly. Perfect and monopolistic competition are characterized by numerous sellers of a product.[120] In a monopoly, however, the market consists only of one seller, while an "[o]ligopoly is an intermediate case . . . with few sellers."[121]

a. Perfect Competition

In order to calculate the profit of a firm in a perfectly competitive market, price, cost, and quantity of a product have to be determined.[122] Beginning with the price, the amount a firm can charge for a product depends on the demand.[123] In case of perfect competition the firm's demand curve is a horizontal line because it only sells a small fraction of the entire market sales.[124] Thus, the quantity it sells will have no effect on the price; in other words, it is a price taker.[125] For example, "[i]f any one of the more than 107,000 soybean farms in the United States were to stop producing soybeans or to double its production, the market price of soybeans would not change appreciably."[126] Thus, the price of a good in a perfectly competitive market is easily determined; it is the market price, P.

Turning to cost and quantity, in general, the greater the quantity a firm produces, the higher the total cost will be. Given a certain quantity and total cost for producing it, it is possible to calculate the firm's average cost, AC, and its marginal cost, MC. The average cost is the firm's total cost divided by the produced quantity.[127] The marginal cost refers to the increase in cost resulting from producing one extra unit in addition to a given

[119] *See Georgia-Pacific*, 318 F. Supp. at 1120 ("The nature and scope of the license, as exclusive or non-exclusive; or as restricted or non-restricted in terms of territory or with respect to whom the manufactured product may be sold.").

[120] EDWIN MANSFIELD & GARY YOHE, MICROECONOMICS: THEORY/APPLICATIONS 289 (11th ed. 2004).

[121] *Id.* at 289. Further market structures exist, however, are not described here. For example, monopsonies and oligopsonies are markets with a single buyer or a few buyers, respectively. *See* PINDYCK & RUBINFELD, *supra* note 109, at 373–81.

[122] *See* MANSFIELD & YOHE, *supra* note 120, at 292, 292 tbl.8.1; PINDYCK & RUBINFELD, *supra* note 109, at 276.

[123] *See* MANSFIELD & YOHE, *supra* note 120, at 291, 291 fig.8.1.

[124] PINDYCK & RUBINFELD, *supra* note 109, at 277–78.

[125] *Id.*

[126] PERLOFF, *supra* note 102, at 243.

[127] PINDYCK & RUBINFELD, *supra* note 109, at 227.

quantity.[128] Similarly as determination of marginal cost, it is also possible to determine marginal revenue, MR. Marginal revenue is the "additional revenue that can be attributed to the sale of one additional unit . . . ."[129]

Now, what quantity should a perfectly competitive firm select to maximize profit? This depends on marginal revenue and marginal cost.[130] Specifically, the firm should choose its quantity so that marginal revenue equals marginal cost.[131] Because in a perfectly competitive market marginal revenue is equal to price, this would also be the quantity where price equals marginal cost. This profit-maximizing quantity is denoted by the point A for a quantity of q^* in figure 5 below.[132] To see that q^* is indeed the profit-maximizing quantity, it can be noted that for a lower quantity, q_1, marginal revenue is greater than marginal cost; thus, profit could be increased by increasing quantity. The shaded area between q_1 and q^* shows the lost profit associated with producing at q_1. At a higher quantity, q_2, however, marginal cost is greater than marginal revenue; thus, reducing quantity saves cost that exceed the reduction in revenue. The shaded area between q^* and q_2 shows the lost profits associated with producing at q_2. Thus, the optimal quantity is at point A.[133] Given price, quantity, and cost the maximum profit of a perfectly competitive firm can be calculated, which is shown by $AB * AC$ in figure 5.

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[128] *Id.*

[129] *See* MANSFIELD & YOHE, *supra* note 120, at 139.

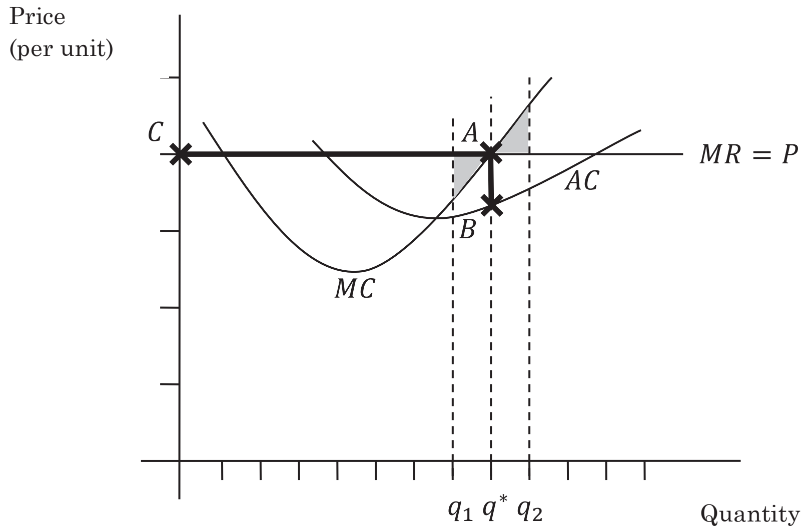
[130] *See* DAVID A. BESANKO & RONALD R. BRAEUTIGAM, MICROECONOMICS 302–304 (2d ed. 2005).

[131] *See* Roger D. Blair & Thomas F. Cotter, *An Economic Analysis of Damages Rules in Intellectual Property Law*, 39 WM. & MARY L. REV. 1585, 1624 (1998) (stating that profit will be maximized where marginal cost is equal to marginal revenue).

[132] PINDYCK & RUBINFELD, *supra* note 109, at 279–80, 280 fig.8.3.

[133] *Id.*

Figure 5



b. Monopoly

Similar as in perfect competition, the profit of a monopolist depends on price, cost, and quantity of a product.[134] However, while in a perfectly competitive market the output quantity of an individual firm does not affect price, in a monopoly market the price of a product increases with decreasing quantity.[135] Thus, the demand curve, D, in a monopoly market is not a horizontal line, as in the perfectly competitive market, but instead has a negative slope.[136] While it can have many forms, in the following it is assumed that there is a linear inverse demand curve defined by the function

$$P = a - bQ, \tag{21}$$

where P is the market price, Q is the total quantity supplied to the market, and a and b are two positive constants representing

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134 *See* ANDREW R. SCHOTTER, MICROECONOMICS: A MODERN APPROACH 297–301 (2d ed. 1997) (stating that the demand and cost can be used to determine the proper price and quantity to produce for the maximum profit in a monopoly).

135 *See* BESANKO & BRAEUTIGAM, *supra* note 130, at 405–06.

136 *See* MANSFIELD & YOHE, *supra* note 120, at 359 (explaining that the demand slope is negative in a monopoly unlike in a perfect competition); *supra* Part III.A.1.a) (stating that the demand curve is a horizontal line in a perfectly competitive market).

all factors that affect price. More specifically, b is the slope of the curve that shows how quantity affects price while a is the intercept of the *y*-axis representing the effects of all factors other than quantity that affect price.[137] Thus, for example, a can encompass "[t]he utility and advantages of the patent property over the old modes or devices, if any, that had been used for working out similar results" as mentioned in *Georgia-Pacific* factor 9 or "[t]he nature of the patented invention; the character of the commercial embodiment of it as owned and produced by the licensor, and the benefits to those who have used the invention" according to *Georgia-Pacific* factor 10.[138]

As described earlier, profit is calculated by price times quantity minus the cost for producing the quantity.[139] Now, given a linear inverse demand function with $P = a - bQ$, the profit of a monopolist can be expressed as

$$\begin{aligned} \Pi &= (a - bQ)Q - cQ \\ \Pi &= (a - c)Q - bQ^2. \end{aligned} \tag{22}$$

In order to calculate the quantity where profit is maximized, marginal revenue and marginal cost have to be considered.[140] In this regard, there is a close relationship between linear inverse demand functions and marginal revenue functions.[141] For any linear inverse demand function of the form $P = a - bQ$, the marginal revenue function has the form $MR = a - 2bQ$.[142] This relationship holds because if price is $P = a - bQ$, it follows that revenue is $r = PQ = aQ - bQ^2$.[143] Taking the derivative with respect to Q, it can be observed that $MR = dR/dQ = a - 2bQ$.[144] Consequently, given the profit of a monopolist in equation (22), it is possible to take its derivative with respect to Q in order to find

137 It should be noted that the constants a and b in equation (21) are used differently than in equation (28), $Q = a - bP$, in which b is the slope of the demand curve that shows how the price of the good affects demand, and a is the intercept of the *y*-axis representing the effects of all factors other than price that affect demand.

138 Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F.Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

139 *See supra* Part III.A.1.

140 *See* DAVID D. FRIEDMAN, PRICE THEORY 217–19 (1986) (demonstrating how marginal revenue is used to calculate the profit-maximizing quantity).

141 *See* RONALD J. VOGEL, PHARMACEUTICAL ECONOMICS AND PUBLIC POLICY 84 (2007) ("The marginal revenue function always has twice the slope of the demand function.").

142 MANSFIELD & YOHE, *supra* note 120, at 141 n.8.

143 *Id.*

144 *See id.*

the quantity at which profit is maximized. Marginal revenue minus marginal cost is

$$MR - MC = (a - c) - 2bQ. \tag{23}$$

Similarly as for the competitive firm, the monopolist's profit would be maximized where marginal revenue equals marginal cost.[145] Thus, it must hold that

$$(a - c) - 2bQ = 0. \tag{24}$$

Solving for Q results in the quantity for which the monopolist will maximize its profit, that is,

$$Q = \frac{(a - c)}{2b}. \tag{25}$$

Plugging the profit-maximizing quantity into $P = a - bQ$, the profit-maximizing price is obtained by

$$P = \frac{(a + c)}{2}.^{146} \tag{26}$$

Plugging the values of cost, price, and quantity into the profit equation (19) will result in the maximum profit of a firm in a monopoly, which is shown by $AB * AC$ in figure 6.[147]

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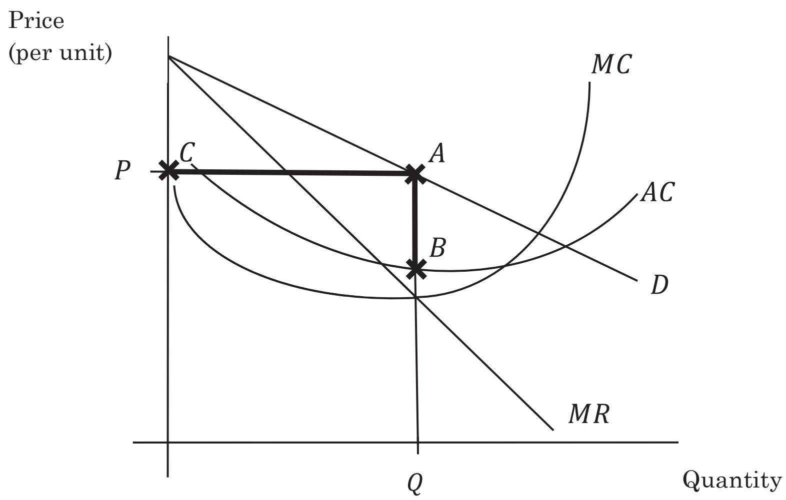
145 *See supra* Part III.A.1.a.

146 *See* MANSFIELD & YOHE, *supra* note 120, at 465 tbl.13.1.

147 *See* PINDYCK & RUBINFELD, *supra* note 109, at 352, 352 fig.10.2 ("[T]o maximize profit, a firm must set output so that marginal revenue is equal to marginal cost.").

Figure 6



c. Monopolistic Competition

Monopolistic competition is a market structure that contains elements of both monopoly and perfect competition.[148] Firms in monopolistic competition compete on selling similar products, which, however, are not perfectly substitutable for one another.[149] For example, shampoo is typically sold in a monopolistically competitive market.[150] In such a market firms can expect to sell more products the larger the total market demand, the fewer the number of firms in the market, and the higher the prices charged by the firm's competitors.[151] These properties of monopolistic competition are captured by the equation

$$q' = Q\left[\frac{1}{n} - b'(p' - \square)\right], \tag{27}$$

where q' is the firm's quantity, Q is the total quantity supplied to the market, n is the number of firms in the market, b' is a constant representing the responsiveness of a firm's sales to its

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148 MANSFIELD & YOHE, *supra* note 120, at 405.
149 PINDYCK & RUBINFELD, *supra* note 109, at 443.
150 *See id.* ("Each firm sells a brand or version of the product that differs in quality, appearance, or reputation, and each firm is the sole producer of its own brand.").
151 *See* PAUL R. KRUGMAN & MAURICE OBSTFELD, INTERNATIONAL ECONOMICS: THEORY & POLICY 121 (8th ed. 2009).

price, p' is the price charged by the firm itself, and P is the average price charged by its competitors.[152] Intuitively, if all firms charge the same price, each will supply an equal quantity of $1/n$. However, a firm charging more than the average price will have a smaller quantity and a firm charging less than the average price will have a larger quantity.[153] It is assumed for equation (27) that total market sales are unaffected by the average price charged by firms in the market, that is, firms can gain market share only at each other's expense, which is not necessarily true in reality.[154]

Similar as in a monopoly, firms in monopolistic competition face negative sloping demand curves because demand for a product decreases when price increases.[155] In fact, the demand function for monopolistic competition (27) is a linear demand function. Given a linear demand curve defined by the function

$$q = a - bp, \tag{28}$$

where q is the firm's quantity, p is the price it charges, b is the slope of the demand curve that shows how the price of the good affects demand, and a is the intercept of the y-axis representing the effects of all factors other than price that affect demand,[156] the demand function for monopolistic competition (27) can be rewritten as

$$q = \left(\frac{Q}{n} + Qb'P\right) - Qb'p, \tag{29}$$

with $(Q/n + Qb'P)$ in place of the constant a and Qb' in place of the constant b.[157]

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152 *Id.*
153 *Id.*
154 *Id.*
155 PINDYCK & RUBINFELD, *supra* note 109, at 445 & fig.12.1.
156 It should be noted that the constants a and b in equation (28) are used differently than in equation (21), $P = a - bQ$, in which b is the slope of the curve that shows how quantity affects price and a is the intercept of the y-axis representing the effects of all factors other than quantity that affect price. *See* BESANKO & BRAEUTIGAM, *supra* note 130, at 41 (explaining the meaning of the constants a and b in the linear demand function $Q = a - bP$).
157 KRUGMAN & OBSTFELD, *supra* note 151, at 122-23.

In order to determine the profit-maximizing quantity, the linear demand function (28) can be rewritten as the linear inverse demand function

$$p = \frac{a - q}{b}. \tag{30}$$

As described earlier, profit is calculated by price times the quantity minus the cost for producing the quantity.[158] Plugging in the linear inverse demand function (30) into the profit equation (19), the result is

$$\begin{gathered} \Pi = \left(\frac{a - q}{b}\right) q - cq \\ \Pi = \frac{aq}{b} - \frac{q^2}{b} - cq. \end{gathered} \tag{31}$$

Then, marginal revenue and marginal cost have to be considered.[159] Taking the derivative with regard to q, marginal revenue minus marginal cost for the monopolistically competitive firm is

$$MR - MC = \frac{a}{b} - \frac{2q}{b} - c. \tag{32}$$

Its profit would be maximized where marginal revenue equals marginal cost. Thus, it must hold that

$$\frac{a}{b} - \frac{2q}{b} - c = 0. \tag{33}$$

Solving for q results in

$$q = \frac{a - bc}{2}. \tag{34}$$

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[158] *Supra* Part III.A.1.
[159] *Supra* Part III.A.1.b.

Plugging in $(Q/n + Qb'P)$ for a and Qb' for b returns the profit-maximizing quantity

$$q = \frac{\frac{Q}{n} + Qb'P - Qb'c}{2}$$

$$q = \frac{\frac{Q}{n} + (P - c)Qb'}{2}. \tag{35}$$

To obtain the profit-maximizing price, the quantity equation (34) is plugged into the price equation (30) resulting in

$$p = \frac{a + bc}{2b}. \tag{36}$$

Then, plugging in $(Q/n + Qb'P)$ for a and Qb' for b results in

$$p = \frac{\frac{Q}{n} + Qb'P + Qb'c}{2Qb'}, \tag{37}$$

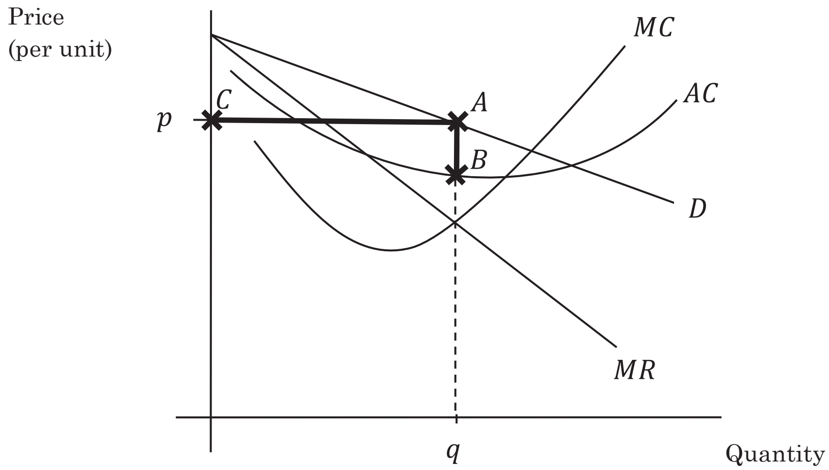
which simplifies to the profit-maximizing price for monopolistic competitors of

$$p = \frac{1}{2nb'} + \frac{(P + c)}{2}. \tag{38}$$

Plugging the values of cost, price, and quantity into the profit equation (19) will result in the maximum profit of a monopolistically competitive firm, which is shown by $AB * AC$ in figure 7.[160]

[160] *See* PINDYCK & RUBINFELD, *supra* note 109, at 445 fig.12.1.

Figure 7



d. Oligopoly

An Oligopoly is a market with only a few firms and with substantial barriers to enter.[161] Because relatively few firms compete in an oligopoly, each firm can influence the price thereby affecting the strategy of its competitors.[162] There are three basic approaches to model an oligopoly: the Cournot, Stackelberg, and Bertrand model.[163] Each approach emphasizes a different aspect of competing in an oligopoly market.[164] In the Cournot model, firms simultaneously choose quantities.[165] In the Stackelberg model, a leader firm chooses its quantity first, after which one or more following firms choose their quantities.[166] In the Bertrand model, instead of choosing quantities, firms simultaneously choose prices.[167] In the following, each model will be addressed in turn.

161 *See* PERLOFF, *supra* note 102, at 445 tbl.13.1.
162 *Id.* at 443.
163 *Id.* at 452.
164 *Id.* at 444.
165 *Id.*
166 *Id.*
167 *Id.*

i. Cournot Competition

In Cournot competition, the profit of a firm depends on the quantity it supplies to the market as well as on the quantity all other firms supply.[168] As described earlier, profit is calculated by price times quantity minus the cost for producing the quantity.[169] Assuming an oligopoly with two firms, where firm 1 produces quantity q_1 and has constant marginal cost c_1, firm 2 produces quantity q_2 and has constant marginal cost c_2, and a linear inverse demand function with $P = a - bQ$, where $Q = q_1 + q_2$, the profit of firm 1 is

$$\begin{gathered} \pi_1 = [a - b(q_1 + q_2)]q_1 - c_1 q_1 \\ \pi_1 = (a - c_1)q_1 - bq_1 q_2 - bq_1^2. \end{gathered} \tag{39}$$[170]

In order to calculate the quantity for maximizing profit, marginal revenue and marginal cost have to be considered.[171] Taking the derivative with regard to q_1, marginal revenue minus marginal cost is

$$MR - MC = (a - c_1) - bq_2 - 2bq_1. \tag{40}$$

Firm 1's profit would be maximized where marginal revenue equals marginal cost. Thus, given the profit-maximizing quantity q_1^*, it must hold that

$$(a - c_1) - bq_2 - 2bq_1^* = 0. \tag{41}$$[172]

Solving for q_1^* results in

$$q_1^* = \frac{a - c_1}{2b} - \frac{1}{2} q_2, \tag{42}$$

168 *Id.* at 453–55, 455 fig.13.2 (explaining how a firm must adjust its output quantity to account for the output quantity of competitors, which in turn affects the firm's marginal revenue and marginal cost calculations that are used in determining profit maximization); JERGER, *supra* note 65, at 55–56 (discussing the calculation of profit maximization in a two-firm oligopoly with the Cournot model).

169 *See supra* Part III.A.1.

170 JERGER, *supra* note 65, at 55.

171 *Supra* Part III.A.1.b.

172 JERGER, *supra* note 65, at 55.

which depends on firm 2's selection of quantity q_2. A similar argument can be made for firm 2 leading to the profit-maximizing quantity q_2^* with

$$q_2^* = \frac{a - c_2}{2b} - \frac{1}{2}q_1, \tag{43}$$

which depends on firm 1's selection of quantity q_1. In order to draw the functions of both firms, this result can be solved for q_1 resulting in

$$q_1 = \frac{a - c_2}{b} - 2q_2^*. \tag{44}$$

Because each firm reacts to the quantity selection of the other firm, their output quantities converge towards a Nash equilibrium, which is the point where neither firm wants to change its strategy, given the strategy of the other firm.[173] As shown in figure 8, at the intersection of the two lines, neither firm has an incentive to deviate from its quantity.[174] Thus, it is a Nash equilibrium.[175]

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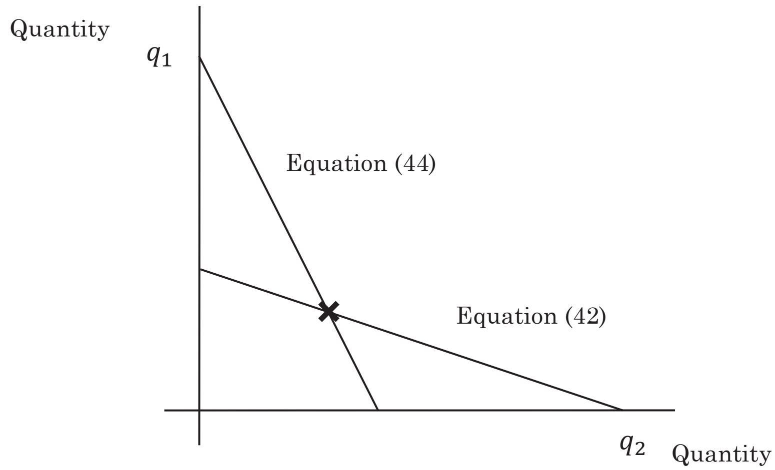
[173] JERGER, *supra* note 65, at 56, 56 fig.3.11; PERLOFF, *supra* note 102, at 456–57, 457 fig.13.3. In a Cournot oligopoly, a Nash equilibrium is also called Cournot-Nash equilibrium or Cournot equilibrium. *See* JERGER, *supra* note 65, at 56 (discussing the origins of the terms).

[174] *Id.* at 56; PERLOFF, *supra* note 102, at 456–57, 457 fig.13.3 (explaining that neither firm in a two-firm oligopoly will opt to change its output quantity level at the intersection of its best-response curves, as the firms will be achieveing profit maximization at those levels).

[175] *See* PINDYCK & RUBINFELD, *supra* note 109, at 454 fig.12.5 (graphically depicting the Nash equilibrium); JERGER, *supra* note 65, at 56 fig.3.11 (describing the Nash equilibrium for a two-firm oligopoly).

Figure 8



Now, at the Nash equilibrium, the profit-maximizing quantity of firm 1 can be calculated. Plugging equation (43) into equation (42) results in

$$q_1^* = \frac{a-c_1}{2b} - \frac{1}{2}q_2$$
$$q_1^* = \frac{a-c_1}{2b} - \frac{1}{2}\left(\frac{a-c_2}{2b} - \frac{1}{2}q_1\right) \tag{45}$$
$$q_1^* = \frac{a-2c_1+c_2}{3b}.$$

Thus, in the symmetric case where $c_1 = c_2 = c$, the profit-maximizing quantity for firm 1 (as well as for firm 2) is one-third of the quantity in a perfectly competitive market; that is,

$$q_1^* = q_2^* = \frac{a-c}{3b}.\text{[176]} \tag{46}$$

To obtain the profit-maximizing price the total quantity $Q = q_1^* + q_2^* = \frac{2}{3}(\frac{a-c}{b})$ is plugged into $P = a - bQ$ resulting in

$$P = \frac{1}{3}(a+2c).\text{[177]} \tag{47}$$

Plugging the values of cost, price, and quantity into the profit

[176] MANSFIELD & YOHE, *supra* note 120, at 432–33.
[177] JERGER, *supra* note 65, at 56.

equation (19) will result in the maximum profit of a firm in Cournot competition.

ii. Stackelberg Competition

Stackelberg competition modifies Cournot competition by accounting for a first mover advantage.[178] Stackelberg competition assumes that one firm is the leader in the market and one or more firms are following.[179] The leader "will [select] a profit-maximizing quantity, taking into [account] the quantity it expects [each] follower to set in reaction to its own [quantity]."[180] Because each follower will want to maximize its profit, the leader can be certain that each follower will accept the leader's quantity choice as given.[181] This assumption permits the leader to predict any follower's quantity and ultimately select its own quantity such that its profit is maximized.[182]

Assuming an oligopoly with two firms and a linear inverse demand function $\square = a - bQ$, the quantities can be computed similarly as the quantities for the Cournot competition.[183] The profit of the leader is

$$\pi_1 = [a - b(q_1 + q_2)]q_1 - c_1 q_1.\text{[184]} \tag{48}$$

Now, plugging the quantity equation of the follower (43) into the profit equation of the leader will result in

$$\begin{gathered} \pi_1 = \left[a - b\left(q_1 + \frac{a - c_2}{2b} - \frac{1}{2}q_1\right)\right]q_1 - c_1 q_1 \\ \pi_1 = \frac{a - c_2}{2}q_1 - \frac{b}{2}q_1^2. \end{gathered} \tag{49}$$

[178] SCHOTTER, *supra* note 134, at 371; PERLOFF, *supra* note 102, at 444.
[179] SCHOTTER, *supra* note 134, at 371.
[180] *Id.*
[181] *Id.*
[182] *Id.*
[183] *Id.*; *see supra* Part III.A.1.d.i.
[184] SCHOTTER, *supra* note 134, at 371.

Taking the derivative with regard to q_1^*, which is the profit-maximizing quantity of the leader, setting it to zero,[185] and solving for q_1^* results in

$$q_1^* = \frac{a-c}{2b},^{186} \tag{50}$$

assuming the symmetric case where $c_1 = c_2 = c$.

The profit-maximizing quantity of the follower, q_2^*, is similarly obtained. Plugging the quantity of the leader into the profit equation of the follower, taking the derivative with regard to q_2^*, setting it to zero, and solving for q_2^* results in

$$q_2^* = \frac{a-c}{4b}.^{187} \tag{51}$$

Comparing these results to the profit-maximizing quantities under Cournot competition, it can be observed that the Stackelberg leader expands its quantity at the expense of the follower's quantity.[188]

To obtain the profit-maximizing price the total quantity $Q = q_1^* + q_2^* = \frac{3}{4}\left(\frac{a-c}{b}\right)$ is plugged into $P = a - bQ$ resulting in

$$P = \frac{1}{4}(a+3c).^{189} \tag{52}$$

Plugging the values of cost, price, and quantity into the profit equation (19) will result in the maximum profit of a firm in Stackelberg competition.[190]

iii. Bertrand Competition

Different from Cournot and Stackelberg competition, Bertrand competition models markets in which firms compete on price rather than on quantity.[191]

> As long as price exceeds marginal cost, an oligopolist in the Bertrand [competition] will always want to undercut [each competitor] by offering a slightly lower price. . . . [A]ssum[ing] that its [competitors] will not meet [the] price cut, it . . . [would] be able

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185 *See supra* Part III.A.1.b.
186 MANSFIELD & YOHE, *supra* note 120, at 465 tbl.13.1.
187 *Id.*
188 *Id.* at 461, 465 tbl.13.1.
189 *See id.* at 465 tbl.13.1.
190 *Id.*
191 *See* STEVEN E. LANDSBURG, PRICE THEORY & APPLICATIONS 389 (8th ed. 2011) (discussing the Bertrand and Cournot models).

> to capture the entire market for itself.
>
> . . .
>
> Bertrand oligopolists will continue to undercut one another until price falls to marginal cost[, at which point] price and [quantity] will be the same [as price and quantity] under . . . [perfect] competition.[192]

Assuming an oligopoly with two firms, both would select their price, which will then determine their quantity. Assuming the symmetric case where marginal costs of firm 1 and 2 are equal, that is, $c_1 = c_2 = c$, the price would be

$$P = c.^{193} \tag{53}$$

The total quantity can be obtained by plugging the marginal cost c into the linear inverse demand function $P = a - bQ$ and solving for Q, which results in

$$Q = \frac{(a-c)}{b}.^{194} \tag{54}$$

If both firms set the same price, the Bertrand model assumes that each firm would sell the same quantity because buyers facing equal prices would be indifferent as to where to buy.[195] Thus, both firms have a probability of 50% of being selected by a buyer.[196] Consequently, the quantity for each firm would be

$$q_1 = q_2 = \frac{(a-c)}{2b}. \tag{55}$$

Plugging the values of cost, price, and quantity into the profit equation (19) will result in the maximum profit of a firm in Bertrand competition.[197]

2. Established Profit

In determining the infringer's anticipated profit, courts often consider the established profit.[198] The established profit performance may provide guidance on the amount of profit the

[192] *Id.*
[193] MANSFIELD & YOHE, *supra* note 120, at 465 tbl.13.1.
[194] *Id.*; *supra* Part III.A.1.b).
[195] MANSFIELD & YOHE, *supra* note 120, at 463.
[196] *Id.*
[197] *See id.* at 465 tbl.13.1.
[198] Trell v. Marlee Elecs. Corp., 912 F.2d 1443, 1446 (Fed. Cir. 1990); Trans-World Mfg. Corp. v. Al Nyman & Sons, Inc., 750 F.2d 1552, 1568 (Fed. Cir. 1984); TERENCE P. ROSS, INTELLECTUAL PROPERTY LAW: DAMAGES AND REMEDIES § 3.08[2][b] (2011).

parties would have estimated during the negotiation.[199] In a similar fashion as *Georgia-Pacific* factor 1 relates to the *established royalty*, *Georgia-Pacific* factor 8 relates to the *established profit*.[200] It permits consideration of "[t]he established profitability of the product made under the patent; its commercial success; and its current popularity."[201] For that purpose, *Georgia-Pacific* factor 11 allows to take account of "[t]he extent to which the infringer has made use of the invention; and any evidence probative of the value of that use."[202]

There is conflicting authority as to whether a reasonable royalty can be higher than the infringer's established profit. On one side, the Federal Circuit suggested "that a reasonable royalty must be fixed so as to leave the infringer a reasonable profit [margin]."[203] "On the other [side], the Federal Circuit [ ] said that '[t]here is no rule that a royalty be no higher than the infringer's net profit margin.'"[204] After all, failure to meet projections does not necessarily imply that they were excessive or based on speculation. Instead, it may simply illustrate the "element of approximation and uncertainty" inherent in projections.[205] Thus, a reasonable royalty can be higher than the infringer's net profit.[206] In that sense, an established profit is admissible as probative of the expectations for the future that the negotiators would have had as of the time of the hypothetical negotiation.[207]

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[199] *See* Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1334 (Fed. Cir. 2009) ("Usage (or similar) data may provide information that the parties would frequently have estimated during the negotiation.").

[200] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

[201] *Id.*

[202] *Id.*

[203] Hughes Tool Co. v. Dresser Indus., Inc., 816 F.2d 1549, 1558 (Fed. Cir. 1987); Grossman, *supra* note 6, at 1420.

[204] Golight, Inc. v. Wal-Mart Stores, Inc., 355 F.3d 1327, 1338 (Fed. Cir. 2004) (quoting State Indus., Inc. v. Mor-Flo Indus., Inc., 883 F.2d 1573, 1580 (Fed. Cir. 1989)); Rite-Hite Corp. v. Kelley Co., 56 F.3d 1538, 1555 (Fed. Cir. 1995); Roy J. Epstein & Alan J. Marcus, *Economic Analysis of the Reasonable Royalty: Simplification and Extension of the Georgia-Pacific Factors*, 85 J. PAT. & TRADEMARK OFF. SOC'Y 555, 559 (2003).

[205] Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1336 (Fed. Cir. 2009) (quoting Unisplay, S.A. v. Am. Elec. Sign Co., 69 F.3d 512, 517 (Fed. Cir. 1995)) (internal quotation marks omitted); Interactive Pictures Corp. v. Infinite Pictures, Inc., 274 F.3d 1371, 1385 (Fed. Cir. 2001) (quoting Unisplay, S.A. v. Am. Elec. Sign Co., 69 F.3d 512, 517 (Fed. Cir. 1995)) (internal quotation marks omitted)).

[206] ROSS, *supra* note 198, at § 3.08[2][b].

[207] Alpex Computer Corp. v. Nintendo Co., 86 Civ. 1749 (KMW), 1994 U.S. Dist. LEXIS 17515, at *134 (S.D.N.Y. Dec. 2, 1994), *rev'd in part*, 102 F.3d 1214

3. Customary Profit

Another factor for determining the infringer's anticipated profit is the customary profit. In this regard, *Georgia-Pacific* factor 12, permits consideration of "[t]he portion of the profit or of the selling price that may be customary in the particular business or in comparable businesses to allow for the use of the invention or analogous inventions."[208] These considerations can be demonstrated by "opinion testimony of qualified experts[,]" as mentioned in *Georgia-Pacific* factor 14.[209] Using a similar language as factor 14, 35 U.S.C. § 284 allows expert testimony on customary profit, specifically stating that "[t]he court may receive expert testimony as an aid to the determination of . . . what royalty would be reasonable under the circumstances."[210]

4. Profit Attributable to the Hypothetical License

If the infringing product is a one-to-one implementation of the hypothetically licensed invention, the profit calculation should be based on the total sales value of the infringing product.[211] However, if the infringing product consists of multiple elements, some of which are not part of the hypothetical license, its value may not be solely based on the infringing element, but also on the noninfringing elements.[212] Accordingly, *Georgia-Pacific* factor 13 limits the profit to "[t]he portion of the realizable profit that should be credited to the invention as distinguished from nonpatented elements, the manufacturing process, business risks, or significant features or improvements added by the infringer."[213] In this regard, "nonpatented" elements refer to all elements of the infringing product beyond the licensed invention—whether actually patented or not.[214]

(Fed. Cir. 1996); CHISUM, *supra* note 8, at § 20.03[3][a], 20.03[3][b][iv]; *see* Peter B. Frank et al., *Patent Infringement Damages*, *in* LITIGATION SERVICES HANDBOOK: THE ROLE OF THE FINANCIAL EXPERT 24-23 (Roman L. Weil et al. eds., 3d ed. 2001) ("The expert might then examine the product's actual profitability and assess the likelihood of the parties projecting such success.").

208 Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

209 *Id.*

210 35 U.S.C.A. § 284 (West, Westlaw through P.L. 112-89 (excluding P.L. 112-55, 112-74, 112-78, and 112-81) approved 1/3/12); *see id.*

211 *See* Brian J. Love, Note, *Patentee Overcompensation and the Entire Market Value Rule*, 60 STAN. L. REV. 263, 268 (2007).

212 *See id.*

213 *Georgia-Pacific*, 318 F. Supp. at 1120.

214 *See* Love, supra note 211, at 268 & n.15.

a. Apportionment

In order to allocate a portion of profit to the infringing element of a multi-element product apportionment is used.[215] However, applying the traditional apportionment approach, that is, identifying the infringing element and determining the profit it generates, can lead to incorrect results. For example, the sum of profits generated by all individual elements may not be identical to the profit generated by the product as a whole.[216] Also, a license may not read on a single element, "but rather on the interface between [elements] or on the way in which the [elements] are assembled and work together."[217] Consequently, even if it covers more than the licensed invention, "[t]here is nothing inherently wrong with using" a multi-element product's entire profit for profit calculation of the infringing element.[218] To the contrary, it is even necessary to base the profit calculation on the profit of the entire product because its market is different from the market of the infringing element.[219]

The amount of profit to be apportioned to the infringing element of a multi-element product can be determined by the Nash bargaining solution. More specifically, the Nash bargaining solution can be extended to n players, each of whom contributed one or more elements to the multi-element product. In such case the unique bargaining solution satisfying the axioms of the Nash bargaining solution is expressed by

$$\max \Omega = \prod_{i=1}^{n} (\pi_i - d_i)\,, \tag{56}$$

[215] Elizabeth M. Bailey et al., *Making Sense of "Apportionment" in Patent Damages*, 12 COLUM. SCI. & TECH. L. REV. 255, 256 (2011); Mark A. Lemley, *Distinguishing Lost Profits from Reasonable Royalties*, 51 WM. & MARY L. REV. 655, 670 (2009).

[216] Bailey et al., *supra* note 215, at 260; Damien Geradin & Anne Layne-Farrar, *Patent Value Apportionment Rules for Complex, Multi-Patent Products*, 27 SANTA CLARA COMPUTER & HIGH TECH. L.J. 763, 773–74 (2011).

[217] *See* Geradin & Layne-Farrar, *supra* note 216, at 774.

[218] Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1338–39 (Fed. Cir. 2009) ("There is nothing inherently wrong with using the market value of the entire product, especially when there is no established market value for the infringing component or feature, so long as the multiplier accounts for the proportion of the base represented by the infringing component or feature."); *see* CAULEY, *supra* note 12, at 111 (discussing the benefit of using the entire product revenues in situations where a component is not sold independently and is not the "basis of consumer demand").

[219] *See* CAULEY, *supra* note 12, at 110–11.

where Π is the total profit generated by the entire product, π_i denotes the payoff that player i can obtain from contributing one or more elements, and d_i denotes its disagreement payoff.[220] Every player contributes a term of $(\pi_i - d_i)$ to the total profit. Thus, given the total profit and the disagreement payoffs of the players, the profit that each player, particularly, the infringer, contributed can be calculated.

b. Entire Market Value Rule

Even if a product consists of multiple elements, its value can be based on one element only.[221] In such case the entire market value rule applies.[222] While initially developed for lost profits calculations, the rule is also used for reasonable royalty calculations.[223] Particularly, it is applicable if the infringing element is the "basis for the customer demand" of the entire product.[224] From the perspective of the Nash bargaining solution this means that the profit from the infringing element is equal to the total profit and the profit from the other elements is zero.[225] For physically separate elements, the entire market value rule requires that the elements constitute a single assembly, parts of a complete machine, or a functional unit.[226] If these requirements are satisfied, the total profit can be allocated to the infringing element.[227] Finally, despite the usual uniform application of the entire market value rule to all buyers of a product, it would also be possible, should a court choose to do so, to limit its application to a subset of buyers as far as the infringing element was the reason for demand for only that subset of buyers.

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[220] OSBORNE & RUBINSTEIN, *supra* note 23, at 23.

[221] Love, *supra* note 211, at 269.

[222] The entire market value rule is less a rule but more an exception to the rule of apportionment. *Id.*

[223] Rite-Hite Corp. v. Kelley Co., 56 F.3d 1538, 1549 (Fed. Cir. 1995).

[224] Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1336 (Fed. Cir. 2009); Bose Corp. v. JBL, Inc., 274 F.3d 1354, 1361 (Fed. Cir. 2001); Fonar Corp. v. Gen. Elec. Co., 107 F.3d 1543, 1552 (Fed. Cir. 1997); *Rite-Hite Corp.*, 56 F.3d at 1549 (quoting State Indus., Inc. v. Mor-Flo Indus., Inc., 883 F.2d 1573, 1580 (Fed. Cir. 1989)). It is insufficient if the infringing element is only a "critical component." *See* Oracle Am., Inc. v. Google, Inc., 798 F.Supp.2d 1111, 1115–16 (N.D. Cal. 2011).

[225] *See* Love, *supra* note 211, at 274–76.

[226] *Rite-Hite Corp.*, 56 F.3d at 1549–50.

[227] *Id.* at 1550.

5. Derivative and Convoyed Sales

Generally, in order to avoid extension of the infringed patent beyond its scope, reasonable royalties cannot cover the sale of noninfringing products.[228] However, courts recognize that, in addition to direct profit from the patented invention, the infringer's anticipated profit can include collateral benefits, which the parties would have taken into account when negotiating a reasonable royalty.[229] After all, "in a hypothetical negotiation [the infringer] would have been more disposed to agree to a high royalty if it could expect to derive such collateral profits[, and,] [c]orrespondingly, [the patentee] . . . would . . . reasonably have demanded [such] higher royalty."[230]

To accommodate for this situation, *Georgia-Pacific* factor 6 allows consideration of "[t]he effect of selling the patented specialty in promoting sales of other products of the licensee; that existing value of the invention to the licensor as a generator of sales of [its] nonpatented items; and the extent of such derivative or convoyed sales."[231] In this regard, "'[c]onvoyed sales' are [sales] made simultaneously with the patented item while 'derivative sales' . . . result [from] the sale of the patented item at a later time."[232] In order to claim royalties for convoyed sales, the "patentee must satisfy the entire market value rule, that is, [it] must prove that the [infringed element] is the basis for customer demand [of the products] to which it seeks to extend its damages."[233] The same is true for derivative sales because the timing of the sale is irrelevant for profit calculation.[234]

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228 A & L Tech. v. Resound Corp., No. C 93-00107 CW, 1995 WL 415146, at *2 (N.D. Cal. June 29, 1995).

229 *See* Trans-World Mfg. Corp. v. Al Nyman & Sons, Inc., 750 F.2d 1552, 1568 (Fed. Cir. 1984) (discussing factors which may be considered when establishing a reasonable royalty); A & L Tech., 1995 WL 415146, at *1–3 (discussing collateral benefits and hypothetical negotiations); ROSS, *supra* note 198, at § 3.08[2][b] (discussing the valuation of a patent).

230 CHISUM, *supra* note 8, at § 20.07[2][f].

231 Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F.Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971).

232 Bensen & White, *supra* note 106, at 55 n.158; Carborundum Co. v. Molten Metal Equip. Innovations, Inc., 72 F.3d 872, 881 n.8 (Fed. Cir. 1995).

233 King Instruments Corp. v. Perego, 65 F.3d 941, 956 (Fed. Cir. 1995) (citing Westinghouse Elec. & Mfg. Co. v. Wagner Elec. & Mfg. Co., 225 U.S. 604, 615 (1912)).

234 *See* Grossman, *supra* note 6, at 1417 ("The patentee may also be able to recover damages from other profits it might have made absent the infringement, such as sales of . . . other items that are related to, but sold separately from, the patented product.").

In order to account for profit generated from convoyed and derivative sales, together called collateral sales, the profit from the collateral sales has to be added to the profit generated by the infringing products.[235] The calculation of the anticipated profit for the collateral sales follows the same model as described above.[236] First the type of the market for the collateral product has to be determined, then revenue from the sale of the collateral product minus the cost for producing it will result in the profit. The profit calculation can be performed for multiple collateral products, which can be finally summed up.[237] Plugging the total sum into the Nash bargaining solution will then return the total profit allocation for infringer and patentee, including collateral sales.[238]

6. Term of the Hypothetical License

The time for determining the amount of reasonable royalties is the point the infringement began.[239] Each party would have been driven by the expectations at this point in time.[240] Thus, profit has to be calculated from that point on for the term of the hypothetical license, that is, the duration of the infringement. Accordingly, *Georgia-Pacific* factor 7, specifies consideration of "[t]he duration of the patent and the term of the license."[241] In the model presented here, the term of the license is implicitly accounted for in the profit-maximization by selecting a quantity over a certain period of time. In this regard, the total quantity over time should be selected according to the term of the hypothetical license. The hypothetical license will run until the end of the infringement or until protection of the right ends, whichever is earlier.

The infringer could try to argue that its economic profit became negative, meaning it had an alternative economic opportunity

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[235] Laura B. Pincus, *The Computation of Damages in Patent Infringement Actions*, 5 HARV. J.L. & TECH. 95, 115 (1991).

[236] *See supra* Part. III.A.1.

[237] *See supra* Part III.A (payoff for collateral products can be based on costs savings as well).

[238] *See supra* Part III.A.4.a) (explaining how different factors of profit can be considered in the Nash bargaining solution).

[239] Applied Med. Res. Corp. v. U.S. Surgical Corp., 435 F.3d 1356, 1363–64 (Fed. Cir. 2006); Panduit Corp. v. Stahlin Bros. Fibre Works, Inc., 575 F.2d 1152, 1158 (6th Cir. 1978); Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *modified by*, 446 F.2d 295 (2d Cir. 1971); ROSS, *supra* note 198, at § 3.08[2].

[240] *See* Frank et al., *supra* note 207, at 24-15 (explaining how in the absence of an agreement the parties rely on expectations).

[241] *Georgia-Pacific*, 318 F. Supp. at 1120.

more lucrative than commercializing the hypothetical license.[242] In such case it would have left the market and instead followed its alternative opportunity realizing the respective disagreement payoff.[243] However, during the term of the infringement it must be assumed that the infringer's economic profit is not negative otherwise it would not have actually remained in the market and instead would have stopped infringing the right and entered another market to realize its disagreement payoff. Thus, the infringer is prohibited from arguing that it would have left the market before the infringement ended.

B. Disagreement Payoffs

In the following, the disagreement payoffs of the patentee and infringer, d_1 and d_2, respectively, are addressed. Generally, the disagreement payoffs cannot result in a higher payoff than the corresponding shares from the total expected payoff because otherwise the negotiations would have been unsuccessful.[244]

1. Opportunity Cost as Disagreement Payoff

The infringer's and patentee's disagreement payoffs are their respective opportunity costs.[245] Opportunity costs, also called economic costs, are "the value[s] of the best alternative use[s] of a resource."[246] Generally, if the infringer's alternative use of a resource is nearly as profitable as the use of the hypothetical license, the patentee could not demand a high royalty.[247] However, if the infringer only has lower value alternatives, the royalty of the patentee would be higher.[248] Similarly, if the alternatives of the patentee are more valuable, the royalties are higher, while they are lower for less valuable alternatives of the

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[242] *See infra* Parts III.B and III.B.1 (explaining opportunity cost).

[243] *See infra* Parts III.B and III.B.1.

[244] *See supra* Part II.A.1 (noting how neither player will accept a result which is worse than its disagreement payoff).

[245] Choi & Weinstein, *supra* note 30, at 57; *See also* Epstein & Marcus, *supra* note 204, at 557 ("The key is to compare the profits from the infringing activity to the profits from the infringer's 'next-best' alternative project at the time of the hypothetical negotiation.").

[246] PERLOFF, *supra* note 102, at 202; *see* Glick, *supra* note 41, at 15 ("Opportunity cost means the benefits that could have been derived from the licensee's next best opportunity.").

[247] *See* Glick, *supra* note 41, at 15 (giving an example of how royalty can be calculated when alternative technologies exist).

[248] Epstein & Marcus, *supra* note 204, at 557–58.

patentee.[249] Based on this relationship between the amount of royalties and the available alternative uses of resources, several courts took account of the opportunity costs when calculating damages.[250]

Addressing the infringer's opportunity cost, if instead of the hypothetical negotiation between the infringer and the patentee, the infringer would have selected a comparable license from a third party, the infringer's opportunity cost is the profit it would have made from commercializing the comparable license.[251] In this sense, *Georgia-Pacific* factor 2, permits consideration of "[t]he rates paid . . . for the use of other patents comparable to the patent in suit."[252] However, if no comparable patent exists and the infringer would also not have pursued a different opportunity of gaining a profit, the opportunity cost would be zero, and so would be the disagreement payoff. Further, it can be observed that the infringer's disagreement payoff cannot be higher than its share of the total expected payoff. Otherwise the infringer would have pursued the alternative business opportunity. Accordingly, the infringer is prevented from arguing that its disagreement payoff would have been higher than its share of the total expected payoff.

For the patentee, the opportunity cost can result from its own commercialization of the right or from licensing it to a third party, for example.[253] Generally, in these cases, the resulting disagreement payoff cannot be higher than what would have been obtained from the infringer because otherwise the patentee would have made a lost profits claim against the infringer or a reasonable royalty claim against a third party.[254] On the other side, the disagreement payoff for the patentee would be zero if it

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[249] *Id.*

[250] *See* Slimfold Mfg. Co., v. Kinkead Indus., Inc. 932 F.2d 1453, 1458 (Fed. Cir. 1991) (discussing available and acceptable noninfringing substitutes and their role in calculating damages); Ellipse Corp. v. Ford Motor Co., 461 F. Supp. 1354, 1368–69 (N.D. Ill. 1978), (discussing a hypothetical negotiation for reasonable royalty determination taking into account alternative technologies), *aff'd*, 614 F.2d, 775 (7th Cir. 1979).

[251] *See* Epstein & Marcus, *supra* note 204, at 558–59 (describing the process an infringer goes through when determining whether or not to license from a patentee).

[252] Georgia-Pacific Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1070), *modified by,* 446 F.2d 295 (2d Cir. 1971).

[253] MARK A. GLICK ET AL., INTELLECTUAL PROPERTY DAMAGES: GUIDELINES AND ANALYSIS 157 (2003).

[254] *See id.* at 157–58, 157 exhibit 7-3 (providing a graph and an example of the boundaries of mutually acceptable royalty rates).

had no opportunity to commercialize the right, which is the case if the patentee had no ability or willingness to license the right or exploit it itself.[255] The disagreement payoff would be also zero if licensing the right to a third party would be an additional licensing transaction on top of the license to the infringer.[256] In such case, the payoff generated from the third party license would not have been forsaken for licensing the infringer, hence, does not represent opportunity cost.

Based on the foregoing, it can be observed that calculation of the disagreement payoff may involve evaluation of another hypothetical negotiation. For example, if the patentee claims that it would have entered into licensing negotiations with a third party, the pertinent licensing negotiation would be evaluated as part of determining the patentee's disagreement payoff.[257] This hypothetical negotiation would be modeled the same way as the hypothetical negotiation between the patentee and the infringer.[258] The patentee's payoff from the negotiation with the third party would then be its disagreement payoff.[259]

2. Patentee's Policy to Preserve the Patent Monopoly

Georgia-Pacific factor 4 takes account of "[t]he licensor's established policy and marketing program to maintain [its] patent monopoly by not licensing others to use the invention or by granting licenses under special conditions designed to preserve that monopoly."[260] Thus, if the patentee excluded licensing the invention, there is no hypothetical negotiation to be modeled, but instead the infringer would pay the patentee's disagreement payoff.[261] If the patentee itself is commercializing

[255] *Cf. id.* at 146 ("In order to be entitled to lost profits damages, the patent owner also must demonstrate that he possesses the marketing, manufacturing, and financing capability to make the infringer's sales.").

[256] *See generally id.* at 156–58 (explaining the range of acceptable royalty rates and the importance of considering opportunity costs).

[257] *See Georgia-Pacific*, 318 F. Supp. at 1121 (describing how the initial inquiry into the hypothetical negotiation of the parties is "often complicated by secondary ones" because the negotiations do "not occur in a vacuum of pure logic").

[258] *See id.* at 1121 (listing a wide range of factors relevant to hypothetical negotiations, including "any other economic factor that normally prudent businessmen would, under similar circumstances, take into consideration in negotiating the hypothetical license.").

[259] *See supra* Part II.A.1 (disagreement payoffs denote the value of pursuing options outside of the hypothetical negotiation).

[260] *Georgia-Pacific*, 318 F. Supp. at 1120.

[261] *See* Panduit Corp. v. Stahlin Bros. Fibre Works, 575 F.2d 1152, 1163 (6th

the patented invention, the disagreement payoff would be the profits lost from being unable to fully exploit the invention.[262] Insofar, the damages calculation is not a reasonable royalty analysis, but rather a lost profit determination.

C. Bargaining Power

It remains to be addressed how the bargaining power of patentee and infringer can be determined. For the infringer, courts will take into account how many noninfringing alternatives equal in terms of cost and performance would have been available on the market.[263] With more substitutes available to the infringer its bargaining position would become stronger.[264] The infringer would be able to make a credible threat to walk away from the negotiations. A similar argument can be made for the patentee. If the patentee would have had other hypothetical licensees apart from the infringer, it could have credibly threatened to stop the negotiations as well. Thus, the possibility of licensing the invention to third parties increases the bargaining power of the patentee.

In this sense, *Georgia-Pacific* factor 5 addresses bargaining power by allowing consideration of "[t]he commercial relationship between the licensor and [infringer], such as, whether they are competitors in the same territory in the same line of business; or whether they are inventor and promoter."[265] Thus, for instance, if patentee and infringer are competitors, the patentee has a possibility to commercialize the technology itself thereby affecting the bargaining power in its favor. However, if the

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Cir. 1978) ("[W]hen the result is to interfere with a patent monopoly, which the patentee was in position to and desired to keep, . . . his compensation for parting against his will with that opportunity must take due account of the loss to him of anticipated profits on the business which the licensees will thus get away from him.").

[262] *See id.* ("[W]hen the patentee's business scheme involves a reasonable expectation of making future profits," it is necessary to consider the anticipated loss of "future business . . . by licensing a competitor to make the machine.").

[263] *See* Zygo Corp. v. Wyko Corp., 79 F.3d 1563, 1571–72 (Fed. Cir. 1996) (discussing a lower courts error in omitting noninfringing alternatives).

[264] *Id.* ("[The infringer] would have been in a stronger position to negotiate for a lower royalty rate knowing it had a competitive noninfringing device [available]."); RICHARD T. RAPP & PHILLIP A. BEUTEL, PATENT DAMAGES: UPDATED RULES ON THE ROAD TO ECONOMIC RATIONALITY 27 (1999), *available at* http://www.nera.com/extImage/3854.pdf ("[I]f there exist good economic substitutes for the patented product or process, the potential licensee has relatively more bargaining power . . . ."); ROSS, *supra* note 198, at § 3.08[2].

[265] *Georgia-Pacific*, 318 F. Supp. at 1120.

patentee is an inventor without the possibility of commercializing the invention, that is, requiring a licensee as promoter, the bargaining power is affected in favor of the infringer.

Generalizing *Georgia-Pacific* factor 5, the bargaining power of patentee and infringer is dependent on the relation between the hypothetical number of licensors and licensees in the market.[266] The bargaining power for the infringer is given by

$$\alpha = \min 1, \left(\frac{\#\ Licensors}{\#\ Licensees}\right), \tag{57}$$

while the infringer is counted as licensee and the patentee is counted as licensor (and also as licensee if it were able and willing to commercialize the invention itself).[267] Given the bargaining power of the infringer, the bargaining power of the patentee is

$$1 - \alpha = 1 - \left[\min 1, \left(\frac{\#\ Licensors}{\#\ Licensees}\right)\right].^{268} \tag{58}$$

Thus, for example, under the assumptions that the patentee is the only licensor for a particular invention, there are four hypothetical licensees, the patentee has the capability and willingness of commercializing the invention, and that it wants to license out one exclusive license, the bargaining power of each licensee is 1/5 and the bargaining power of the patentee is 4/5. However, assuming another example of two licensors, both of which are not capable or willing to commercialize the invention, and only one licensee, that is, the infringer, the bargaining power of the infringer will be 1, while the bargaining power of the licensors is 0, respectively.

IV. OUTCOME OF THE HYPOTHETICAL NEGOTIATION

Having determined the total expected payoff, disagreement payoffs, and bargaining powers, the Nash bargaining solution

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[266] *See id.*; RAPP & BEUTEL, *supra* note 264, at 27 ("If the patentee has alternative licensees . . . the patentee can credibly threaten to walk away from the bargaining table and will, therefore, obtain the better deal.").

[267] *See generally* Jarosz & Chapman, *supra* note 42, at 261–62 (explaining how licensing prospects contribute to unequal relative bargaining power).

[268] Adjusting the equation based on the assumption that the bargaining power of the infringer and the patentee together is equal to 1 therefore either's individual bargaining power is equal to 1 minus the others bargaining power.

determines the patentee's and infringer's respective payoffs. The result is based on various assumptions that may have to be modified to fit the individual case. Beyond the assumptions already discussed,[269] the hypothetical negotiation also assumes perfect exchange of information,[270] and that the patent is valid, enforceable, and infringed.[271] However, these assumptions do not invalidate the reasonable royalty determination.[272] "While the [amount of] damages may not be based on mere speculation or conjecture, it will be sufficient . . . to prove [its] extent . . . 'as a matter of just and reasonable inference' . . . ."[273] Sufficiency of this approximation "is based, in part, on [ ] recognition of the artificial construct of the hypothetical negotiation that is used to determine the reasonable royalty."[274] However, courts require that the game-theoretic methodology is also sufficiently connected to the facts of the case.[275] The patentee must prove its

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[269] *See supra* Part III.A ("[A]ssum[ing] that the infringement occurs due to selling an infringing product . . . ."); *supra* Part III.A.1 ("[A]ssum[ing] that the infringer wants to maximize profit whenever possible . . . ."), Part III.A.1.b (assumption of a linear inverse demand function); *supra* Part III.A.1.c (discussing the assumption of a linear demand function); III.A.1.d (discussing the assumption of an oligopoly with two firms and that marginal costs of all firms are equal).

[270] Proctor & Gamble Co. v. Paragon Trade Brands, Inc., 989 F. Supp. 547, 606 (D. Del. 1997) (discussing hypothetical negotiations and that each party would have all relevant information); CAULEY, *supra* note 12, at 26.

[271] *See* Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1325 (Fed. Cir. 2009) ("The hypothetical negotiation also assumes that the asserted patent claims are valid and infringed."); *Proctor & Gamble Co.*, 989 F. Supp. at 606; PARR & SMITH, *supra* note 13, at 653.

[272] *Lucent Techs., Inc.*, 580 F.3d at 1336 (Fed. Cir. 2009) ("[J]udges must scrutinize the evidence carefully to ensure that the 'substantial evidence' standard is satisfied, while keeping in mind that a reasonable royalty analysis 'necessarily involves an element of approximation and uncertainty.'") (quoting Unisplay, S.A. v. Amer. Elec. Sign Co., 69 F.3d 512, 517 (Fed. Cir. 1995); Del Mar Avionics, Inc. v. Quinton Instrument Co., 836 F.2d 1320, 1327–28 (Fed. Cir. 1987) ("The determination of a damage award is not an exact science . . . . [I]t will be appropriate for the court to consider all factors reasonably pertinent to a determination of damages that bear a reasonable relationship to the damages actually suffered by the patent owner."); Faulkner v. Gibbs, 199 F.2d 635, 639–40 (9th Cir. 1952) ("[T]he loss can only be determined by reasonable approximation.").

[273] ROSS, *supra* note 198, at § 3.08 (quoting Story Parchment Co. v. Paterson Parchment Paper Co., 282 U.S. 555, 563 (1931).

[274] *Id.* at § 3.08.

[275] Mformation Techs., Inc. v. Research in Motion Ltd., No. C 08–04990 JW, 2012 WL 1142537, at *2–3 (N.D. Cal. Mar. 29, 2012) ("[T]he Court finds that [the expert's] royalty rate calculation [using the Nash bargaining solution] is sufficiently tied to the facts of the case."); Oracle Am., Inc. v. Google, Inc., 798 F.Supp.2d 1111, 1119 (N.D. Cal. 2011) ("Because [the expert] did not provide an adequate justification for applying the Nash solution to the facts of this case,

approximation by a preponderance of the evidence.[276]

A royalty can take several forms, such as a percentage of profit, a fixed sum per unit, or a lump sum.[277] The model presented in this article expresses the royalty, that is, the patentee's share of profit π_1, as a percentage of profit over the period of the infringement, which, however, can be easily converted into a per-unit or per-amount royalty. For finding the per-unit royalty of the patentee's share, π_1 can be expressed as

$$\pi_1 = rQ, \tag{59}$$

where r is the per-unit royalty rate and Q is the quantity of units sold.[278] Solving for r, results in a per-unit royalty of

$$r = \frac{\pi_1}{Q}. \tag{60}$$

Similarly, a per-dollar royalty rate can be expressed by

$$\pi_1 = rPQ, \tag{61}$$

where P is the price per unit.[279] Solving for r, results in a per-dollar royalty of

$$r = \frac{\pi_1}{PQ}. \tag{62}$$

V. SUMMARY OF THE PRESENTED MODEL

This article has described how game-theoretic bargaining solutions and microeconomic market models can be combined to model a hypothetical reasonable royalty negotiation in accordance with 35 U.S.C. § 284 as interpreted by *Georgia-Pacific.* The framework of the hypothetical negotiation under *Georgia-Pacific* factor 15 can either be based on a cooperative or noncooperative bargaining solution because both approaches

allowing him to testify to his results would risk misleading the jury as to the soundness of the foundation for his conclusions.").

276 SmithKline Diagnostics, Inc. v. Helena Labs. Corp., 926 F.2d 1161, 1164 (Fed. Cir. 1991); SUNG, *supra* note 115, at 304.

277 *See* Vincent A. Thomas et al., *Reasonable Royalty as a Measure of Damages in Patent Infringement Matters*, *in* ECONOMIC DAMAGES IN INTELLECTUAL PROPERTY: A HANDS-ON GUIDE TO LITIGATION 172 (Daniel Slottje ed., 2006) (discussing different ways royalties can be negotiated).

278 *See* Choi & Weinstein, *supra* note 30, at 57–59 (explaining equations to solve for per-unit royalties); *see also* Jarosz & Chapman, *supra* note 42, at 253.

279 Jarosz & Chapman, *supra* note 42, at 256–57.

complement each other and can lead to the same result. When selecting the Nash bargaining solution, the reasonable royalty depends on three values: the total expected payoff, the disagreement payoff, and the bargaining power of each player. Determining these three values implies correlating the remaining *Georgia-Pacific* factors 1 through 14 to the structure of the Nash bargaining solution.

The total expected payoff is the profit the infringer anticipated from commercialization of the infringed intellectual property right. In order to determine the anticipated profit, the infringer's established profit and the customary profit can provide useful guidance. Most importantly, however, the anticipated profit depends on the market in which a product is sold. Markets can be categorized according to the number of firms supplying a product and the customers' ability to distinguish between the products. The infringer will maximize its profit in a market by supplying a quantity such that marginal revenue equals marginal cost. Profit further depends on the profit-maximizing price and cost for producing a product. The anticipated profit obtained this way can be adjusted for multi-element products by using the Nash bargaining solution to determine how much profit the infringing element contributed. Then, calculation of the anticipated profit can be finalized by adding profit generated from collateral sales, if any.

The disagreement payoffs of the infringer and patentee are their respective opportunity costs, that is, the values of their best alternative uses of resources. In this regard, it can be observed that the infringer is prevented from arguing that its disagreement payoff would have been higher than its share of the total expected payoff because such claim would contradict its actions of infringing the patent, which is based on the anticipation of generating the highest profit among all available business alternatives. Also, the disagreement payoff of the patentee cannot be higher than its share from the total expected payoff because otherwise the patentee would not have made an infringement claim against the infringer in the first place. If the patentee excluded licensing the invention, no negotiation would have taken place; hence, the infringer is required to pay the patentee's disagreement payoff, that is, lost profits, if any. Generally, calculation of the disagreement payoff can involve evaluation of another hypothetical negotiation.

For the bargaining power of patentee and infringer, it should be taken into account whether any noninfringing alternatives equal to the infringed rights in terms of cost and performance would have been available to the infringer. With more substitutes available the bargaining position of the infringer would become stronger. Conversely, with fewer substitutes available the bargaining position of the patentee would become stronger. Similarly, the bargaining positions are affected by the number of other hypothetical licensees the patentee could license apart from the infringer.

Having assessed total expected payoff, disagreement payoffs, and bargaining power of the patentee and infringer, the Nash bargaining solution can be used to calculate a reasonable royalty. If necessary, the result can be converted to a per-unit royalty, per-dollar royalty, or other form.